INDUCTION I

1. Fill in the missing steps of the following proof.

Theorem. $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$ *for every* $n \in \mathbb{N}$.

Proof (by induction on n). Base case: $n = 1$. Then

Inductive step. Suppose $k \in \mathbb{N}$ and $1 + 2 + 4 + \cdots + 2^k = 2^{k+1} - 1$. Then

$$1 + 2 + 4 + \cdots + 2^{k+1} = \left(1 + 2 + 4 + \cdots + 2^k\right) + 2^{k+1}$$

$$=$$

Therefore, by induction, $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$ for every $n \in \mathbb{N}$. □

2. Use induction (on n) to prove that for all $n \in \mathbb{N}$, $3^n \geq 1 + 2^n$.

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3. Use induction (on n) to prove de Moivre's formula: for all $n \in \mathbb{N}$ and all $x \in \mathbb{R}$

$$\boxed{(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)}$$

(where $i = \sqrt{-1}$). You might need to use the following trig identities:

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

Challenge. Prove that for all $n \in \mathbb{N}$, $8 \mid (5^{2n} - 1)$.