PORTFOLIO PROOFS E

Instructions. Choose one of the following statements and prove it (using induction). Use LaTeX to write your proof nicely. Drop your proof (both pdf and tex) in your OneDrive folder by the end of the day Friday, November 19.

1. Let $n \in \mathbb{N}$. If $n \geq 12$, then there are non-negative integers a and b such that $n = 4a + 5b$.

2. Consider the 2×2 matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. For any $n \in \mathbb{N}$, $A^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$ (where F_k is the k^{th} term of the Fibonacci sequence in definition 1).

3. Define a new function on the positive real numbers: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Prove that if $n \in \mathbb{N}$, then $\Gamma(n+1) = n!$. (This makes Γ a version of the factorial that is defined for non-integers; for example, $\Gamma(1/2) = \sqrt{\pi}$).

Definition 1. The Fibonacci sequence is defined recursively by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for any $n \in \mathbb{N}$. The beginning of the Fibonacci sequence is $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$.

Date: November 12, 2021.