LOGIC

1. Make truth tables for the statements $P \vee (\neg P)$ and $P \wedge (\neg P)$.

Definition. A statement is a **contradiction** if its only possible truth value is False. A statement is a **tautology** if its only possible truth value is True.

2. Fill in the blanks with a statement (written in English) that makes the entire statement true.

a) $[P \wedge (\neg P)] \implies$ ______________________________

b) ______________________________ $\implies [P \vee (\neg P)]$

3. Make a truth table for the statement $P \implies (P \vee Q)$.

4. Make a truth table for the statement $[(P \implies Q) \wedge P] \implies Q$ (this is known as Modus Ponens).

5. The statement in problem 3 might be expressed in English as "If P is true, then we know that either P or Q (or both) must be true." Produce a similar English version of Modus Ponens (the statement in problem 4).

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6. Consider the statement, "In 2018 Dr. Axon gave an A to any student who got a perfect score on the final exam."

a) Express this as a statement of the form $P \implies Q$.

b) Suppose Adrian got a perfect score on the final exam. What can you conclude about Adrian's grade in the class? (You're almost certainly using Modus Ponens to make this deduction).

c) Blair didn't get an A in Prof. Axon's class. What can you conclude about Blair's score on the final exam?

d) You probably just used Modus Tollens, another very important tautology. Apply the same reasoning to fill in the blanks below to get Modus Tollens ...

...in English:

If P implies Q, and we know that Q is not true, then ________________________________

...in symbols:

$$[(P \implies Q) \wedge (\neg Q)] \implies \_\_\_\_\_\_\_\_$$

Definition. The **converse** of $P \implies Q$ is $Q \implies P$. The **contrapositive** of $P \implies Q$ is $(\neg Q) \implies (\neg P)$.

7. Write a true statement that ...

a) ...has a true converse.

b) ...has a false converse.

c) ...has a true contrapositive.

d) ...has a false contrapositive.[1]

[1]Or use a truth table to show that $P \implies Q$ is logically equivalent to its contrapositive