INDUCTION I

1. Prove that for any $n \in \mathbb{N}$ and any sets $A_1, A_2, \ldots, A_n$,

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n}.$$

2. Find the flaw in the following proof.

Proposition. *For any $n \in \mathbb{N}$ and any lines $l_1, l_2, \ldots, l_n$ in $\mathbb{R}^2$, the intersection $l_1 \cap l_2 \cap \cdots \cap l_n$ is either $\emptyset$ or l_i for some $i \in \{1, 2, \ldots, n\}$.*

Proof by induction. Base case: $n = 1$. This means we have just one line l_1. Clearly $l_1 = l_1$, which establishes the base case.

Now suppose that $k \in \mathbb{N}$ and that for any lines $l_1, l_2, \ldots, l_k$ in $\mathbb{R}^2$, $l_1 \cap l_2 \cap \cdots \cap l_k$ is either $\emptyset$ or l_i for some $i \in \{1, 2, \ldots, k\}$. Let $l_1, l_2, \ldots, l_{k+1}$ be lines in $\mathbb{R}^2$. If $l_1 \cap l_2 \cap \cdots \cap l_{k+1} = \emptyset$, then we're done. If $l_1 \cap l_2 \cap \cdots \cap l_{k+1} \neq \emptyset$, then $l_1 \cap l_2 \cap \cdots \cap l_k \neq \emptyset$. Then by the inductive hypothesis, $l_1 \cap l_2 \cap \cdots \cap l_k = l_i$ for some $i \in \{1, 2, \ldots, k\}$. Fix this i so we can continue to work with this particular line l_i. Then we have $\emptyset \neq l_1 \cap l_2 \cap \cdots \cap l_k \cap l_{k+1} = l_i \cap l_{k+1} = l_i \cap l_i \cap \cdots \cap l_i \cap l_{k+1}$ where l_i is repeated $k-1$ times. This means we can again apply the inductive hypotheses to conclude that $l_i \cap l_{k+1}$ is either l_i or l_{k+1}. This suffices to establish the inductive step. Therefore the proposition holds for every $n \in \mathbb{N}$.

□

Date: November 5, 2021.