INSTRUCTIONS: Answer **three of questions 1–4**, **two of questions 5–7**, and **one of questions 8 and 9** now (this is **six problems total**). Write your solutions on the provided sheets of blank paper. Double check to make sure your name is on the front and your solutions are clearly labeled, then staple your solutions together and turn them in. Take this page with you and **solve the remaining 3 problems over the break**. Turn these problems in on Wednesday, April 8.

As always, write proofs (and disproofs) clearly using English words, grammar, and punctuation. You may refer to the following definitions and propositions (as well as any others that you remember). Calculators, phones, books, or other outside materials are not allowed on the in-class portion of the exam. You may use any non-living materials you like for the take-home portion of the exam (books are okay, but people aren't). If you find a solution somewhere, please cite your source.

Definition 1. Let $a \in \mathbb{Z}$. Then a is *even* if there is an integer n such that $a = 2n$ and a is *odd* if there is an integer n such that $a = 2n + 1$.

Definition 2. Let $a, b \in \mathbb{Z}$. We say that a *divides* b (written $a|b$) if there is an integer n such that $an = b$.

Definition 3. A natural number $n \geq 2$ is *prime* if its only positive divisors are 1 and n.

Definition 4. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. Then $a \equiv b \pmod{n}$ if $n|(a - b)$.

Proposition 1. *Let $a, b \in \mathbb{Z}$ and $p \in \mathbb{N}$. If p is prime and $p|ab$, then $p|a$ or $p|b$.*

Answer three of questions 1–4.

1. Let a, b, and c be integers. Prove that if $a|b$ and $b|c$, then $a|c$.

2. Prove that there are no integers m and n for which $24m + 10n = 3$.

3. Let $a, b \in \mathbb{Z}$. Prove that if $a + b$ is even, then a and b have the same parity.

4. Prove that for any sets A and B, if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

Answer two of questions 5–7.

5. Determine if the following statement is true or false. Prove it if it is true or provide a counterexample if it is false.

Let $n, p \in \mathbb{N}$. If p is prime, then $\gcd(n, p) = 1$.

6. Determine if the following statement is true or false. Prove it if it is true or provide a counterexample if it is false.

For any $a \in \mathbb{Z}$, if a^2 is not divisible by 4, then a is odd.

7. Determine if the following statement is true or false. Prove it if it is true or provide a disproof if it is false.

There are prime numbers p and q such that $p \equiv q \pmod{4}$.

Answer one of the following questions. Both can be proved using mathematical induction.

8. Prove that for every $n \in \mathbb{N}$, $1 + 3 + 9 + \cdots + 3^n = \dfrac{3^{n+1} - 1}{2}$.

9. Prove that for every natural number n that is greater than or equal to 4, $n! \geq 2^n$.