Definition 1. Integers a and b have the same *parity* if a and b are both odd or both even.

Definition 2. Let a and b be integers and n be a natural number. Then $a \equiv b \pmod{n}$ if $n|(a-b)$.

INSTRUCTIONS: Prove the following. All proofs should either be direct or by contraposition. You may use any propositions from the book, homework, or class that you find helpful; if you do so, please refer to the relevant result clearly (e.g. "as proved in class, if a^2 is even, then a is even").

1. Let $a, b \in \mathbb{Z}$. If ab is even, then a or b is even.

2. Let $a, b \in \mathbb{Z}$. If $a + b$ is even, then a and b have the same parity.

3. Let $a, b \in \mathbb{Z}$. If $a + b$ is even and ab is even, then a and b are both even.

4. Let $x \in \mathbb{Z}$. If 4 does not divide x^2, then x is odd.

5. Let $x \in \mathbb{Z}$. If x is odd, then $x^2 - 1 \equiv 0 \pmod{8}$.

6. For any $a, b \in \mathbb{Z}$, it follows that $(a+b)^3 \equiv a^3 + b^3 \pmod{3}$.

7. Let $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.