NAME(S):

MATH 301 LOGIC JANUARY 28, 2015

1. Determine if the following statements are True or False.

a) Either horses have 4 legs or 17 is not prime.

b) Neither do 3 quarters add up to one dollar, nor do horses have 4 legs.

c) If ducks have webbed feet, then Canada lies south of the equator.

d) If Canada lies south of the equator, then ducks are mammals.

Definition 1. A statement is a *contradiction* if its only possible truth value is "false". A statement is a *tautology* if its only possible truth value is "true".

2. Make a truth table for $P \vee (\neg P)$ and $P \wedge (\neg P)$.

3. Fill in the blank with a statement (written in English) that makes the entire statement true.

a) $[P \wedge (\neg P)] \implies$ ____

b) ____ $\implies [P \vee (\neg P)]$

4. Make a truth table for $P \implies (P \vee Q)$.

5. Make a truth table for $[(P \implies Q) \wedge P] \implies Q$ (this is known as Modus Ponens and is one of the most important rules of deductive logic).

6. Last semester Prof. Axon gave an A to any student who got a perfect score on the final exam.

a) Katie got a perfect score on the final exam. What can you conclude about Katie's grade in the class? (You're almost certainly using Modus Ponens to make this deduction).

b) Mike didn't get an A in Prof. Axon's class. What can you conclude about Mike's score on the final exam?

b) You probably just used Modus Tollens, another very important tautology. Apply the same reasoning to fill in the blanks below to get a useful rule of logic. Modus Tollens in English:

If we know that P implies Q and we know that Q is not true, then ______________________________

Modus Tollens in symbols:

$$[(P \implies Q) \wedge (\neg Q)] \implies \underline{\hspace{4cm}}$$

Definition 2. The *converse* of $P \implies Q$ is $Q \implies P$. The *contrapositive* of $P \implies Q$ is $(\neg Q) \implies (\neg P)$.

7. a) Write a statement that has a true contrapositive.

b) Try to think of a statement that has a false contrapositive (if you can't do this, use a truth table to show that $P \implies Q$ is logically equivalent to its contrapositive).

8. We have seen that $P \implies Q$ is logically equivalent to $(\neg P) \vee Q$. One of DeMorgan's laws states that $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$. Use this to find an alternative expression for $\neg(P \implies Q)$.