Name(s):
Math 301 First Proofs February 6, 2015

Definition. Let $n, m \in \mathbb{Z}$.

1. n is *even* if there is $a \in \mathbb{Z}$ such that $n = 2a$.
2. n is *odd* if there is $a \in \mathbb{Z}$ such that $n = 2a + 1$.
3. m *divides* n (written $m|n$) if there is $a \in \mathbb{Z}$ such that $ma = n$.
4. If $n \geq 2$ and the only positive divisors n are 1 and n, then n is *prime*.
5. If $n \geq 2$ and n is not prime, then n is *composite*.

1. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|c$, then $a|(b + c)$.

2. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|(b + c)$, then $a|c$.

3. Let $x, y \in \mathbb{Z}$.

a) Prove that if x or y is even, then xy is even.

b) State the converse of "if x or y is even, then xy is even". Note that the converse has the same implicit quantifier(s) as the original statement.

c) Is the converse true or false? (There is no need to prove it).

d) It is also true that if $6|x$ or $6|y$, then $6|(xy)$. Is the converse of this statement true?

4. Prove that every odd integer is the difference of two squares (e.g. $5 = 3^2 - 2^2$).