Name(s):

Math 301 — Contradictions and Biconditionals — February 25, 2015

Lemma 1. *Let $n, p \in \mathbb{N}$ with p prime. Then for any integer a it follows that $p|a$ if and only if $p|a^n$.*

Recall that a biconditional statement like this is really two implications:

1. If $p|a$, then $p|a^n$;
2. If $p|a^n$, then $p|a$.

Proving the lemma requires two proofs, one for each of these implications. Each implication can be proved using any of our proof techniques: direct, contrapositive, or contradiction. For now we won't prove the lemma (the first implication is easy to prove directly but the second is more complicated) but you may use it anyway.

1. Prove that $\sqrt[3]{3}$ is irrational.

2. Let $a \in \mathbb{Z}$. Prove that a is even if and only if a^3 is even.

3. Let $a \in \mathbb{Z}$. Prove that $6|a$ if and only if $2|a$ and $3|a$.