NAME(S):
MATH 301 — INDUCTION MINI-HOMEWORK — MARCH 23, 2015

1. Find the flaw(s) in the following induction proof.

Proposition. *If $n \in \mathbb{N}$, then $n+1 \leq n$.*

Proof by induction. Suppose that $k+1 \leq k$. Then $k+2 = (k+1)+1 \leq k+1$ by hypothesis. Therefore by induction $n+1 \leq n$ for all natural numbers n. □

2. Use induction to prove that the following proposition. Some of the proof is already done–you only need to fill in the blank.

Proposition. *For any natural number n,* $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

Proof by induction. Base case: $n = 1$. Then $1^2 = \frac{1(2)(3)}{6}$.
Inductive step: prove that if $1^2 + 2^2 + 3^2 + \cdots + k^2 = \dfrac{k(k+1)(2k+1)}{6}$, then $1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = \dfrac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$. Suppose that $1^2 + 2^2 + 3^2 + \cdots + k^2 = \dfrac{k(k+1)(2k+1)}{6}$.

Therefore $1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = \dfrac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$. □

3. Let n be an integer greater than or equal to 2. Use induction to prove that for any n sets $A_1, A_2, A_3, \ldots, A_n$ the following holds:

$$\overline{A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_4} = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \cdots \cap \overline{A_n}.$$

4. Determine if the statement is true and provide a proof or disproof: $1 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$ for any natural number n. Hint: it may be helpful to remember that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.