1. Find the flaw(s) in the following induction proof.

Proposition. *All dogs are the same color.*

Proof by induction. The proposition may be interpreted as saying that every set of n dogs contains dogs of only one color. We procede by induction on the number n.

Base case: $n = 1$. This means that we have a set of just one dog. Obviously, if there's just one dog, then all dogs in the set are the same color.

Iductive step: let $k \in \mathbb{N}$ and suppose that any set of k dogs consists of dogs of just one color. Let A be a set of $k+1$ dogs. Let a be one of the dogs in A. Then the set $A - \{a\}$ is a set of k dogs and hence they are all the same color (by the inductive hypothesis). Let b be another dog in A. Similarly, $A - \{b\}$ is a set of k dogs and thus all of these dogs are the same color. Because $a \in A - \{b\}$ and $b \in A - \{a\}$ it follows that dog a and dog b are the same color as all the other dogs in A. Hence A consists of dogs of only one color.

Therefore, by induction, all dogs are the same color. □

2. Use induction to prove that for any non-negative integer n, any set with n elements has 2^n subsets.

3. Prove that for any $n \in \mathbb{N}$, $1(2) + 2(3) + 3(4) + \cdots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$.

4. Prove that for any $n \in \mathbb{N}$, $\dfrac{1}{1} + \dfrac{1}{4} + \dfrac{1}{9} + \cdots + \dfrac{1}{n^2} \leq 2 - \dfrac{1}{n}$.