Name(s):

Math 301 Pigeonholes and Functions April 20, 2015

1. Prove that if each point in the plane $\mathbb{R}^2$ is colored either red or blue, then there must be two points of the same color exactly one unit apart.

2. In each square of a 5×5 board there is a flea. At some point, all the fleas jump to an adjacent square (two squares are adjacent if they share an edge). Is it possible that after they settle in the new squares, the configuration is exactly as before: one flea per square? Hint: suppose the board is colored like a chess board; how many squares of each color are there?

Definition 1. Let A, B, and C be sets and let $f : A \to B$ and $g : B \to C$ be functions. The composite function $g \circ f : A \to C$ is defined by $(g \circ f)(x) = g(f(x))$.

3. Prove that if f and g are both injective, then the composite function $g \circ f$ is injective.

4. Prove that if f and g are both surjective, then the composite function $g \circ f$ is surjective.

5. Writing as little as possible, prove that if f and g are both bijective, then $g \circ f$ is bijective.