INSTRUCTIONS: Solve two of these problems for your final portfolio entry and write them nicely along with one other problem from homework, exam 2, or a worksheet. Your final portfolio should be one PDF document consisting of 8 solutions. See the syllabus and section 5.3 of The Book of Proof for some suggestions and guidelines for writing. The final portfolio is due by email by 12:00 pm (noon) on Friday, May 8.

Definition 1. A figure in the plane is *convex* if for any two points in the figure, the line segment connecting the two points lies within the figure.

1. A well known theorem of geometry states that the sum of the interior angles of a triangle must be $180°$.

a) Determine the sum of the interior angles of any convex quadrilateral.

b) Determine the sum of the interior angles of any convex pentagon.

c) Let n be a natural number greater than 2. Make a conjecture about the sum of the interior angles of a convex figure with n sides. Prove your conjecture.

2. Prove that for any $n \in \mathbb{N}$, a set with n elements has $\frac{n(n-1)}{2}$ two-element subsets.

3. Let $n \in \mathbb{N}$ and $a, b \in \mathbb{Z}$. Prove that if $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$ for every $k \in \mathbb{N}$.

4. Prove that the sum of the cubes of any 3 consecutive natural numbers is divisible by 9.

5. For which natural numbers n is $n^2 < 2^n$? Prove that you are correct.

6. Consider the equation

$$x^5 + x^4 + x^3 + x^2 + 1 = 0 \tag{1}$$

a) Prove that equation 1 has at least one real solution.

b) Prove that equation 1 has no rational solution.

7. The game of Chomp is played on an $m \times n$ array of squares with the top left square missing (an example of a possible board is shown below). Two players take turns picking a square and removing it along with all squares below it, to the right of it, or both. The player who chomps the last square wins. Prove that the player who plays first must have a winning strategy regardless of the dimensions of the board. Hint: there is a simple non-constructive proof (and no one seems to know the actual winning strategy).