Proposition 1. *Let $x \in \mathbb{R}$. If $x > 0$, then $x + \dfrac{1}{x} \geq 2$.*

Definition 1. suppose that **f** and **g** are functions such that the two limits $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist, and that **k** is a constant. Then,

1) $\lim\limits_{x \to a} k = k$
2) $\lim\limits_{x \to a} f(x) + g(x) = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$

Proof. Let $x \in \mathbb{R}$ and suppose $x + \frac{1}{x} < 2$. Then we can take the limit as x approaches $-\infty$ of both sides to get: $\lim\limits_{x \to -\infty} x + \dfrac{1}{x} < \lim\limits_{x \to -\infty} 2$. Using Definition 1, this can be rewritten as $x + 0 < 2$ or simply $x < 2$. Hence, x must be less than 2 in order for this proposition to hold. Since $0 < 2$, we can therefore conclude that $x \leq 0$. □

Comments:

Proposition 1. *Let $x \in \mathbb{R}$. If $x > 0$, then $x + \dfrac{1}{x} \geq 2$.*

Proof. Let $x \in \mathbb{R}$.

Assume x is positive and $x \neq 0$.

Then, $x + \frac{1}{x} \geq 2$

$x^2 + 1 \geq 2x$

$x^2 - 2x + 1 \geq 0$

$(x-1)^2 \geq 0$

Thus, the left hand side will always be greater than or equal to 0.

Therefore, $x + \frac{1}{x} \geq 2$. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and suppose a is odd. By definition there exists $c \in \mathbb{Z}$ such that $a = 2c + 1$. Hence $a^2 - 1 = (2c+1)^2 - 1 = (4c^2+4c+1) - 1 = 4c^2+4c$. Then $a^2 - 1 = 4c(c+1)$. And $c(c+1)$ is the product of two consecutive integers and is therefore even. Let $c(c+1) = 2p$, $p \in \mathbb{Z}$. Thus $a^2 - 1 = 4(2p) = 8p$, $p \in \mathbb{Z}$. Therefore by definition of divides, $8|(a^2 - 1)$. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.
Proof. Let $a \in \mathbb{Z}$ and suppose a is odd.
Thus, there is an integer b such that $a = 2b + 1$ by definition of an odd number.
By definition of divisibility, there is an integer c such that $8c = (a^2 - 1)$.
Then, by substitution, $8c = ((2b + 1)^2 - 1)$, $8c = ((4b^2 + 4b + 1) - 1)$, $8c = 4b^2 + 4b$, $4(2c) = 4(b^2 + b)$.
Thus, $8 \mid (a^2 - 1)$.

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and also let a be odd. By definition $\exists b \in \mathbb{Z}, a = 2b + 1$. By definition, $\exists m \in \mathbb{Z}, b = 2m$. Then, $b + 1 = 2m + 1$. Then, $c = ((2m)(2m + 1))/2 = (4n^2 + 2n)/2 = 2(2n^2 + n)/2 = 2n^2 + n$. Since $2n^2 + n$ is an integer, we conclude, $8|(a^2 - 1)$. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and suppose a is odd. □

Proof. By definition (of divides), there is $\exists n \in \mathbb{Z}$ such that $8n = a^2 - 1$. □

Proof. Then, by definition (of odd), there is $\exists m \in \mathbb{Z}$ such that $a = 2m + 1$. □

Proof. Substitute a into $8n = a^2 - 1 \equiv 8n = (2m + 1)^2 + 1$. □

Proof. $8n = 4m^2 + 4m + 1 - 1$ □

Proof. $8n = 4(m^2 + m)$ □

Proof. WLOG, Suppose m is even.
Then by definition (of even), there is $\exists x \in \mathbb{Z}$ such that $m = 2x$ □

Proof. $8n = 4((2x)^2 + 2x)$ □

Proof. $8n = 4(4x^2 + 2x)$ □

Proof. $8n = 8(2x^2 + x)$ where $2x^2 + x \in \mathbb{Z}$ □

Proof. Thus, $8 \mid (a^2 - 1)$ □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and suppose a is odd. By definition (of odd) there is an integer n such that $a = 2n + 1$. Then $a^2 - 1 = (2n + 1)^2 - 1 = 4n^2 + 4n + 1 - 1 = 4(n^2 + n)$. Hence $a^2 - 1 = 4(n^2 + n)$. By definition (of divides) there is an integer m such that $8m = a^2 - 1$. Then $8m = a^2 - 1 = 4(n^2 + n)$. Consequently $2m = n^2 + n$. Thus $2m = n^2 + n$ and $n^2 + n \in \mathbb{Z}$. Therefore $8 \mid (a^2 - 1)$. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Suppose a is odd, so a=2n+1 for some integer n. Then $(a^2 - 1) = (2n + 1)^2 - 1$ which equals $(4n^2 + 4n)$ which equals $(4(n^2 + n))$ which equals 4n(n+1). So, now we have $(a^2 - 1) = 4n(n + 1)$. To get that factor of 8, we can see that either n or n+1 are even, so n(n+1) must be even. From this, we can have n(n+1)=2b for some integer b. Therefore, we have $(a^2 - 1 = 4n(n + 1))$ which equals 4(2b)=8b. Therefore, $(8b = n^2 - 1)$ which by the definition of divides means $(8|(n^2 - 1))$.

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and suppose a is odd.
By definition, $\exists n \in \mathbb{Z}$ such that $2n + 1 = a$.
Now, $a^2 - 1 = (2n+1)^2 - 1 = 4n^2 + 4n + 1 - 1 = 4n^2 + 4n = 4(n^2) + 4(n)$. Since $n^2, n \in \mathbb{Z}$, $4|n^2$ and $4|n$.
By definition, $\exists x \in \mathbb{Z}$ such that $4x = n$.
So $a^2 - 1 = 4(n^2) + 4(n) = 4(4x)^2 + 4(4x) = 64x^2 + 16x = 8(8x^2 + 2x)$, where $(8x^2 + 2x) \in \mathbb{Z}$.
Therefore, $8|(a^2 - 1)$. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Definitions:
Definition 1: a is odd if there is $c \in \mathbb{Z}$, such that $a = 2c + 1$.
Definition 2: a divides b, written $a|b$, if there is $c \in \mathbb{Z}$, such that $mc = a$.
Proposition 1.
Suppose $a \in \mathbb{Z}$, if a is odd, then $8|(a^2 - 1)$.
Proof. Let $a \in \mathbb{Z}$ and suppose a is odd.
By definition of odd, there is an integer c, such that $a = 2c + 1$.
By definition of divides, $a|b$, if there is an interger c, such that $mc = a$.
Then, when substituted, $a^2 - 1 = (2c+1)^2 - 1 = 4c^2 + 4c = 8(1/2c^2 + 1/2c)$.
Where $1/2c^2 + 1/2c$ is an integer.
Therefore, $8|(a^2 - 1)$.

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Let $a \in \mathbb{Z}$ and suppose a is odd. By definition, there is $m \in \mathbb{Z}$ such that $a = 2m + 1$. Then, we see that $8|(a^2 - 1) = 8|((2m + 1)^2 - 1) = 8|4m^2 + 4m = 8|4(m^2 + m)$. We can also see that $m^2 + m$ is an even number because $2|m^2$. Therefore, because $m^2 + m \in \mathbb{Z}$, we can conclude that $8|a^2 - 1$ □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof:
Let a $\in \mathbb{Z}$. Suppose a is odd.
By definition, c $\in \mathbb{Z}$ such that $a = 2c + 1$
Then, $(a^2 - 1) = (2c + 1)^2 - 1 = 4c^2 + 4c + 1 - 1 = 4c^2 + 4c = 2(2c^2 + 2c)$

Comments:

Proposition 2. *Suppose* $a \in \mathbb{Z}$. *If* a *is odd, then* $8 \mid (a^2 - 1)$.

Suppose a is an element of a set of integers and assume a is odd.
By definition, of odd, there is an integer c such that $a = 2c + 1$..
Thus, $a^2 - 1 = (2c + 1)^2 - 1$..
This is equivalent to $4c^2 + 4c$, which equals $4c(c + 1)$..
So far, $a^2 - 1 = 4c(c + 1)$, but we want a factor of 8 instead of 4.
Notice that c or $c + 1$ must be even in order for $c(c + 1)$ to be even.
Hence, $c(c + 1) = 2k$, for some integer k.
Now, $a^2 - 1 = 4c(c + 1)$, which equals $4(2k) = 8k$..
$a^2 - 1 = 8k$ is the same as $8|(a^2 - 1)$. Thus, the original statement holds.

Comments:

Proposition 2. *Suppose* $a \in \mathbb{Z}$. *If* a *is odd, then* $8 \mid (a^2 - 1)$.

Proof. Let $a \in \mathbb{Z}$ and suppose that a is odd. By definition $a = 2c + 1$ where $c \in \mathbb{Z}$.

$$\begin{aligned} a^2 - 1 &= (2c + 1)^2 - 1 \\ &= 4c^2 + 4c + 1 - 1 \\ &= 4c^2 + 4c \\ &= 4(c^2 + c) \end{aligned}$$

Case 1. If c is even, $c^2 + c$ is also even. By definition $c = 2n$ where $n \in \mathbb{Z}$. $(2n)^2 + 2n = 4n^2 + 2n = 2(n^2 + n)$ where $(n^2 + n)$ is an integer. Therefore $c^2 + c$ is even by definition.

Case 2. If c is odd then $c^2 + c$ is even. By definition $c = 2n + 1$ where $n \in \mathbb{Z}$. $c^2 + c = (2n + 1)^2 + (2n + 1) = (4n^2 + 4n + 1) + (2n + 1) = 4n^2 + 6n + 2 = 2(2n^2 + 3n + 1)$ Where $(2n^2 + 3n + 1)$ is an integer.
Thus $c^2 + c = 2(2n^2 + 3n + 1)$ and therefore $c^2 + c$ is even by definition.

Since $c^2 + c$ is even whether c is odd or even, then $2|(c^2 + c)$. Plugging this into $4(c^2) + c$ we conclude that $8|4(c^2 + c)$ and therefore $8|a^2 - 1$ if a is odd. □

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof 1: Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.
Contrapositive: If 8 does not divide $mid(a^2 - 1)$, then a is even.
By definition there exists some $c \in \mathbb{Z}$ such that $8c \neq\mid (a^2 - 1)$.
$8c + 1 \neq a^2$
$2(4c + 1) = a^2$
Thus, a^2 is even. By definition there exists some $c \in \mathbb{Z}$ such that $a^2 = 2c$
$2 \mid (a^2)$. $2 \mid (a)$. Thus a is even.
This proves the contrapositive.

Comments:

Proposition 2. *Suppose $a \in \mathbb{Z}$. If a is odd, then $8 \mid (a^2 - 1)$.*

Proof. Suppose $a \in \mathbb{Z}$ such that a is odd. By definition, there is $m \in \mathbb{Z}$ such that $a = 2m + 1$. Then,

$$\begin{aligned} a^2 - 1 &= (2m+1)^2 - 1 \\ &= 4m^2 + 4m + 1 - 1 \\ &= 8\left(\frac{m^2 + m}{2}\right) = a^2 - 1 \end{aligned} \tag{1}$$

Therefore $8|(a^2 - 1)$ □

Comments: