MATH 301 PORTFOLIO

YOUR NAME

1. Expository entries

A. **Response to TITLE by AUTHOR.** Paste your first portfolio entry here.

B. **Response to TITLE by AUTHOR.** Paste your second portfolio entry here.

C. **Response to WHATEVER YOU DID FOR THE THIRD EXPOSITORY PORTFOLIO.** Paste your third portfolio entry here.

2. Proofs

A. **Direct and contrapositive proofs.** Paste your first proof here. I recommend putting your proof in the following format.

If you wish to include a relevant definition, use a definition environment:

Definition 1. $\lim_{x\to a} f(x) = L$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon.$$

State the result you proved as a theorem, then give the proof. If you'd like to include a lemma, then use a lemma environment.

Lemma. $\lim_{x\to 0} f(x) \neq 0$ *if there is a number* $\epsilon > 0$ *such that for every number* $\delta > 0$ *there is a number* x *such that*

$$0 < |x| < \delta \textit{ and } |f(x)| \geq \epsilon.$$

Proof. This is the negation of the definition of the limit (definition 1) with $a = 0$ and $L = 0$. All quantifiers have been swapped, even the implicit quantifier for the conditional statement at the end of the definition. □

Theorem. $\lim\limits_{x\to 0} \sin(1/x) \neq 0.$

Proof. Let δ be a real number greater than 0. Let n be an odd natural number such that $\frac{1}{n} < \delta$. Because $0 < \frac{2}{\pi} < 1$, it follows that $0 < \frac{2}{n\pi} < \delta$. Moreover, because n is odd

$$\sin\left(\frac{1}{(2/(n\pi))}\right) \;=\; \sin\left(\frac{n\pi}{2}\right) \;= 1.$$

Thus we have shown that there is a number $x = \frac{2}{n\pi}$, such that

$$0 < |x| < \delta \text{ and } |\sin(1/x)| \geq 1.$$

Since δ was arbitrary, we can conclude that this holds for every positive real number δ. Therefore, by the lemma, $\lim\limits_{x\to 0} \sin(1/x) \neq 0$ □

B. **Proofs by contradiction and non-conditional statements.** Paste your second proof here.

Date: December 1, 2018.

C. **Proofs and disproofs.** Paste your third proof (or disproof) here. It may not be be appropriate to call the statement in question a theorem, since you may be providing a disproof. I suggest the following.

Statement. *Suppose A, B, and C are sets. If $A \times C \subseteq B \times C$, then $A \subseteq B$.*

The statement is **false**.

Proof. To prove that the statement is false, we produce sets A, B, and C, such that $A \times C \subseteq B \times C$ and $A \nsubseteq B$. Et cetera. $\square$

D. **Induction I.** Paste your fourth proof here.

E. **Induction II.** Paste your fifth proof here.

F. **Uncategorized proofs.** Paste your sixth proof here.