LOGIC

1. Determine if each of the following statements is true or false.

a) Either horses have 4 legs or 17 is not prime.
b) Neither do 3 quarters add up to one dollar, nor do horses have 4 legs.
c) If ducks have webbed feet, then Canada lies south of the equator.
d) If Canada lies south of the equator, then ducks are mammals.

2. Make truth tables for the statements $P \vee (\neg P)$ and $P \wedge (\neg P)$.

Definition. A statement is a **contradiction** if its only possible truth value is false. A statement is a **tautology** if its only possible truth value is true.

3. Fill in the blank with a statement (written in normal English, like "ducks are mammals") that makes the entire statement true (no matter what P is).

a) $[P \wedge (\neg P)] \implies$ _____

b) _____ $\implies [P \vee (\neg P)]$

4. Make a truth table for the statement $P \implies (P \vee Q)$.

5. Make a truth table for the statement $[(P \implies Q) \wedge P] \implies Q$.

6. The statement in problem 4 might be expressed in English as "If P is true, then we know that either P or Q (or both) must be true." Produce a similar English version of the statement in problem 5.[1]

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[1]The tautology in problem 5 gives a logical rule called Modus Ponens.

7. Consider the statement, "In 2018 Dr. Axon gave an A to any student who got a perfect score on the final exam."

a) Express this as a statement of the form $P \implies Q$.

b) Suppose Adrian got a perfect score on the final exam. What can you conclude about Adrian's grade in the class?[2]

c) Blair didn't get an A in Prof. Axon's class. What can you conclude about Blair's score on the final exam?

d) You probably just used Modus Tollens, another very important tautology. Apply the same reasoning to fill in the blanks below to get Modus Tollens ...

... in English: If P implies Q, and Q is not true, then _____

... in symbols: $[(P \implies Q) \wedge (\neg Q)] \implies$ _____

Definition. The **converse** of $P \implies Q$ is $Q \implies P$. The **contrapositive** of $P \implies Q$ is $(\neg Q) \implies (\neg P)$.

8. Write a true statement (in English) that ...

a) ... has a true converse.

b) ... has a false converse.

c) ... has a true contrapositive.

d) ... has a false contrapositive.[3]

[2] You almost certainly used Modus Ponens to make this deduction.

[3] Or use a truth table to show that $P \implies Q$ is logically equivalent to its contrapositive