CONTRADICTION AND CONTRAPOSITIVE PROOFS

Proposition 1. *Let $a, b \in \mathbb{Z}$. If 7 does not divide ab, then 7 divides neither a nor b.*

1. Write the **contrapositive** and the **negation** of proposition 1.

Two proofs of proposition 1 are given below. Both proofs basically work, but neither is perfect. Read the proofs carefully and note places where they could be improved. Then write a better proof of the theorem (using whatever approach you deem best).

> *Proof.* Suppose $a, b \in \mathbb{Z}$ and $7|a$ or $7|b$. By definition $7n = a$ or $7n = b$. Then $7na$ or $7nb = ab$. By definition $7|(ab)$. This proves the contrapositive. □

> *Proof by contradiction.* Suppose $7 \nmid (ab)$ and $7|a$ or $7|b$. WLOG, suppose $7|a$. Then $ab = 7nb$ for some $n \in \mathbb{Z}$. This is a contradiction. □

2. Write a better proof of proposition 1.

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3. Prove that for every $n \in \mathbb{N}$, $4 \nmid (n^2 + 2)$.

Challenge. Prove that $\log_2 3$ is irrational.