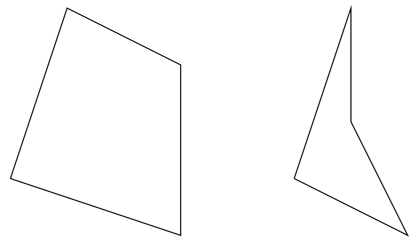
INDUCTION II ... AND RELATIONS

1. A well-known geometry theorem states that this sum of the interior angles of a triangle must be $180°$. This problem guides you through finding a similar formula for any convex polygon (with $3, 4, 5, \ldots$ sides). Convex means that every interior angle is less than $180°$; the first quadrilateral below is convex, but the second isn't.[1]



a) Determine the sum of the interior angles of a convex quadrilateral (hint: divide it into two triangles).

b) Determine the sum of the interior angles of a convex pentagon.

c) Determine the sum of the interior angles of a convex hexagon. Hint: divide it into a triangle and a pentagon.

d) Generalize to find a formula for the sum of interior angles of a convex polygon with n sides (where $n \geq 3$). Prove your formula is correct.

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[1]Technically, convex means that the straight line between any two points of the polygon is entirely inside the polygon

Definition. Let R be a relation on the set A.

a) R is **reflexive** if $\forall x \in A,\ xRx$.
b) R is **symmetric** if $\forall x, y \in A,\ xRy \implies yRx$.
c) R is **transitive** if $\forall x, y, z \in A,\ (xRy \wedge yRz) \implies xRz$.

2. Which of the three properties in the definition does the relation of $<$ (on $\mathbb{R}$) have?

3. Which of the three properties in the definition does the relation of divides (on $\mathbb{Z}$) have?

4. Which of the three properties in the definition does the relation of congruence modulo 5 (on $\mathbb{Z}$) have?

Challenge. Prove that you are correct in problems 3–5.