Definition. A random variable X has a **discrete uniform distribution** if it is equally likely to assume any one of a finite set of possible values.

Examples. Flip a fair coin. Roll a single (fair) die. Choose a winning number in a lottery.

Definition. A random variable X has a **Bernoulli distribution** with parameter θ (with $0 < \theta < 1$) if its probability mass function is

$$\boxed{m(x) = \begin{cases} 1-\theta & \text{if } x = 0 \\ \theta & \text{if } x = 1 \end{cases}}$$

The outcome 1 is often referred to as "success" while 0 is "failure" and the experiment is often called a Bernoulli trial.

Proposition. *The mean and variance of a Bernoulli random variable are $\mu = \theta$ and $\sigma^2 = \theta(1-\theta)$.*

Examples. Flip a biased coin once. Ask one person a yes or no question. Play the lottery once.

Definition. The total number of successes in n independent, identically distributed (iid) Bernoulli trials is a random variable with a **Binomial distribution**. The probabilty mass function of a random variable X having a binomial distribution with parameters n and θ is

$$\boxed{b(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x} \text{ for } x = 0, 1, \ldots, n}$$

Proposition. *The mean and variance of a binomial distribution are $\mu = n\theta$ and $\sigma^2 = n\theta(1-\theta)$.*

Examples. Flip n identical biased coins and count the number of heads. Ask n randomly selected people a yes or no question.

Definition. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed (iid) Bernoulli trials, all with probability of success θ. Let N be the trial on which the first success occurs. The random variable N is said to have a **geometric distribution** with parameter θ and its probability mass function is

$$\boxed{g(n) = \theta(1-\theta)^{n-1} \text{ for } n = 1, 2, 3, \ldots}$$

Proposition. *The mean and variance of a geometric distribution are $\mu = \frac{1}{\theta}$ and $\sigma^2 = \frac{1}{\theta}\left(\frac{1}{\theta} - 1\right)$.*

Examples. Flip a biased coin until the first heads appears. Roll a pair of dice until you first get a pair of sixes. Ask randomly selected people a yes or no question until you first get a yes.

Definition. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed (iid) Bernoulli random variables, all with probability of success θ. Let N be the trial on which the k^{th} success occurs (so the possible values for N are $k, k+1, k+2, \ldots$). The random variable N is said to have a **negative binomial (or binomial waiting-time or Pascal) distribution** with parameters k and θ and its probability mass function is

$$\boxed{m(n) = \binom{n-1}{k-1}\theta^k(1-\theta)^{n-k} \text{ for } n = k, k+1, k+2, \ldots}$$

Proposition. *The mean and variance of a negative binomial distribution are* $\mu = \dfrac{k}{\theta}$ *and* $\sigma^2 = \dfrac{k}{\theta}\left(\dfrac{1}{\theta} - 1\right)$.

Definition. A random variable with the probability mass function

$$\boxed{p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \ldots}$$

is said to have a **Poisson distribution** with parameter $\lambda > 0$.

Proposition. *The mean and variance of a Poisson distribution are* $\mu = \lambda$ *and* $\sigma^2 = \lambda$.

Definition. Suppose n elements are to be selected without replacement from a population of size N of which M are successes. The number of successes selected is a **hypergeometric** random variable and its probability distribution function is

$$\boxed{h(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}}$$

Proposition. *The mean and variance of a hypergeometric distribution are* $\mu = \dfrac{nM}{N}$ *and*
$\sigma^2 = \dfrac{nM(N-M)(N-n)}{N^2(N-1)}$.