Math 321 **Exam 1** **September 27, 2019**

Instructions: Solve $N-1$ of the following N problems and write your solutions on the provided paper, clearly labeling each solution (do not write your solutions on this sheet). All solutions should include a clear method or argument and should use English words and sentences when appropriate. Clear and comprehensible solutions will generally earn more points than those that are hard to understand; a correct solution without supporting work may receive little or no credit.

Indicate which problem you are skipping by placing an **X** in the corresponding box below. Leave the rest blank (I'll use them to record your scores). Calculators, phones, and all other devices are forbidden. Answers may be left unsimplified.

Name:								
1	2	3	4	5	6	...	N	Total

Method. The number of ways to select k elements from an n-element set is...

	Order matters	Order doesn't matter
With replacement	n^k	$\binom{n+k-1}{k}$
Without replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k} = \frac{n!}{(n-k)!k!}$

Thoerem. *Properties of (all) probabilities:*

1. $P(\emptyset) = 0$
2. $P(A) = 1 - P\left(A^C\right)$
3. *If* $A \subseteq B$, *then* $P(A) \leq P(B)$
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Definition. Let A and B be events with $P(B) \neq 0$. The **conditional probability of A given B** is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Thoerem (Multiplication rule for probabilities)**.** *Let A and B be events with $P(B) \neq 0$. Then*

$$P(A \cap B) = P(A|B)P(B)$$

Thoerem (The Law of Total Probability)**.** *If event B has probability strictly between 0 and 1, then*

$$P(A) = P(A|B)P(B) + P(A|B^C)P(B^C)$$

Thoerem (Bayes' Law)**.** *If A and B are events with positive probability, then*

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Definition. The **expected value** (or **mean**) of a random variable is a kind of weighted average and is denoted $E(X)$ or μ.

1. If X is a discrete RV with PMF $p(x)$, then $E(X) = \sum_x xp(x)$.
2. If X is a continuous RV with PDF $f(x)$, then $E(X) = \int_{-\infty}^{\infty} xf(x)dx$.