

INSTRUCTIONS: Solve any 9 of the following 10 problems and write your solutions on the provided paper, clearly labeling each solution. Write your solutions clearly and use English words and sentences to explain your work where appropriate. Even a correct solution may receive little or no credit if a method of solution is not shown. You may use a calculator during the exam, but you are not required to use one: answers like “The probability of being dealt two pair is $\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$,” are acceptable. You may not use a phone, notes, your book, or any other resources.

FORMULAS: $E(X) = \sum_x xf(x)$ $Var(X) = E(X^2) - [E(X)]^2$ Bayes' Rule: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

The Law of Total Probability (short version): $P(B) = P(B|A)P(A) + P(B|A')P(A')$.

Bernoulli distribution with parameter θ : pdf $f(x) = \begin{cases} \theta & \text{if } x = 1 \\ 1 - \theta & \text{if } x = 0 \end{cases}$, mean $\mu = \theta$, variance $\sigma^2 = \theta(1 - \theta)$.

Geometric distribution with parameter θ : pdf $f(x) = \theta(1 - \theta)^{x-1}$ for $x = 1, 2, 3, \dots$, mean $\mu = \frac{1}{\theta}$, variance $\sigma^2 = \frac{1-\theta}{\theta^2}$.

Binomial distribution with parameters θ and n : pdf $f(x) = \binom{n}{x}\theta^x(1 - \theta)^{n-x}$ for $x = 0, 1, 2, \dots, n$, mean $\mu = n\theta$, variance $\sigma^2 = n\theta(1 - \theta)$.

PROBLEMS:

1. Suppose events A and B are independent and $P(A) = 0.4$ and $P(B) = 0.5$. Calculate $P(A \cup B)$.
2. In a blind taste test a person is asked to rank, in order of preference, colas A, B, C, and D.
 - a) How many possible rankings are there?
 - b) Assuming that there is no difference in flavor between the colas, what is the probability that the taster ranks cola A first?
3. An experiment consists of flipping a fair coin 3 times. Let A be the event that you flip an odd number of heads. Let B be the event that you flip 2 or more heads. Are A and B independent events? Explain your answer or justify it using calculations.
4. You have 6 socks in a drawer (individual socks, not pairs), 4 of which are gray, and 2 of which are blue. If you pick 2 socks at random, what is the probability that you end up with a matching pair?
5. Two students miss an exam. They tell their teacher that they missed the test because of a flat tire. The teacher gives them a make-up exam, the last question of which is “Which tire was flat?” Assuming that there wasn't really a flat tire and the students each choose one of the 4 tires at random, what is the probability that the students both give the same answer?
6. John Stockton's career playoff free throw percentage is 81%. For the purposes of this problem, assume this means he makes each free throw with probability 0.81, independently of all other free throw attempts. Let X be the number of free throws Stockton attempts before he misses one (including the missed free throw in the count). Calculate the expected value of X .
7. John Stockton's career playoff free throw percentage is 81%. For the purposes of this problem, assume this means he makes each free throw with probability 0.81, independently of all other free throw attempts. Suppose that Stockton attempts 12 free throws in one game. Let Y be the number of free throws Stockton makes in that game. Calculate $P(Y \leq 10)$.
8. A random variable X has the probability distribution function (pdf) given below. Calculate the variance of X .

$$f(x) = \begin{cases} \frac{2}{5} & \text{if } x = 0 \\ \frac{1}{5} & \text{if } x = 1 \\ \frac{2}{5} & \text{if } x = 2 \end{cases}$$

9. The winning times for the men's marathon at all the Olympics since 1964 are given below (the format is HH:MM:SS). Sketch a reasonably accurate box and whisker plot of the data. Mark any outliers (those points more than $1.5f_s$ from the nearest fourth) with separate dots.

2:06:32 2:08:01 2:09:21 2:09:55 2:10:11 2:10:32 2:10:55 2:11:03 2:12:11 2:12:19 2:12:36 2:13:23 2:20:26

10. A recent poll of 400 likely South Carolina Republican primary voters by American Research Group, Inc. found that 40% were under 50. Of those voters who were under 50, 35% supported Trump. Of those voters who were over 50, only 32% supported Trump. What percentage of Trump supporters are under 50?