COUNTING

Axioms of Probability. *The assignment of probability to events in a sample space S must obey the following rules:*

(1) $P(E) \geq 0$ *for any event* E

(2) $P(S) = 1$

(3) If $E_1, E_2, E_3, \dots$ *are disjoint events, then* $P(E_1 \cup E_2 \cup E_3 \cup \dots) = \sum_{i=1}^{\infty} P(E_i)$

1. In the experiment of flipping a coin twice we found a sample space consisting of 4 equally likely outcomes:

$$S = \{HH, HT, TH, TT\}$$

a) Are the events $\{HH\}$, $\{HT\}$, $\{TH\}$, and $\{TT\}$ disjoint?

b) What is the union of the events $\{HH\}$, $\{HT\}$, $\{TH\}$, and $\{TT\}$?

c) Use axioms 2 and 3 to find a number to finish the equation

$$P(\{HH\}) + P(\{HT\}) + P(\{TH\}) + P(\{TT\}) =$$

d) What is the relationship between the probabilities of the events $\{HH\}$, $\{HT\}$, $\{TH\}$, and $\{TT\}$?

e) Find the probability of each outcome.

Note. Writing $P(\{HH\})$ is kind of annoying, so when we're dealing with simple events (as opposed to compound events), we'll usually leave off the curly braces and just write $P(HH)$.

2. Generalize from the first problem. Let n be a positive integer and suppose the sample space for an experiment consists of n equally likely outcomes.

a) The probability of any one outcome is ...

b) The probability of event E is ...

Date: September 6, 2019.

Method. (Multiplication rule for counting) If a process occurs in two steps and there are m options for the first step and n options for the second, then there are mn total possibilities.

3. A PIN consists of four of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, each of which may be used any number of times (selection with replacement, order matters).

a) How many PINs start with 123?

b) How many PINs start with 12?

c) How many total PINs are there?

d) What is the probability that a randomly selected PIN starts with 123?

e) What is the probability that a randomly selected PIN starts with 12?

4. Continue working with 4-digit PINs.

a) How many different PINs are there if each digit can be used at most once (selection without replacement, order matters)?

b) How many different PINs use the digits 1234?

c) How many different PINs are there if each digit can be used at most once and the order doesn't matter (selection without replacement, order doesn't matter)? Hint: in part b you calculated how many times different orderings of 1234 appeared in your enumeration of PINs in part a. You can use these two numbers to answer this question.

5. Our original goal was to analyze the Match 4 lottery in which four numbers are chosen from 1 to 24 without replacement and order doesn't matter.

a) How many possible outcomes are there?

b) How many do not match any of my chosen numbers 01 02 03 04?

c) How many match exactly one of my chosen numbers? Hint: think of this as a two-step process, with the first step being choosing one of my numbers to match.