CONDITIONAL PROBABILITY AND INDEPENDENCE

The die-coin experiment consists of rolling a (normal, 6-sided) die and then flipping a fair coin the number of times shown on the die. For example, if you roll a 1 you'll flip the coin once, but if you roll a 2 you'll flip the coin twice. A sample space for this experiment is

$$S = \{(1, H), (1, T), (2, HH), (2, HT), (2, TH), (2, TT), \ldots, (6, TTTTTT)\}$$

(this time the outcomes are not equally likely).

We'll deal with two random variables associated with this experiment:

- the number rolled on the die: R
- the number of times we flip heads: F.

Our goal is to calculate the PMF for F.

1. What are the possible values for R and what is the probability of R taking each of those values?

2. What are the possible values for F? (Do not attempt to calculate probabilities yet).

Calculating the probabilities for F is difficult unless we are given information about the roll of the die. For example, if we know that $R = 1$, then we know that $F = 0$ with probability $\frac{1}{2}$. This is a **conditional probability** and is expressed symbolically as $P(F = 0|R = 1) = \frac{1}{2}$ (read as "the probability of $F = 0$ given $R = 1$").

3. The conditional probabilities in this problem are all given $R = 2$, that is you rolled a two and are thus flipping the coin twice. Use this to calculate the following:

a) Calculate $P(F = 0|R = 2)$

b) Calculate $P(F = 1|R = 2)$

c) Calculate $P(F = 2|R = 2)$

Date: September 11, 2019.

Theorem (The Law of Total Probability)**.** *If event B has probability strictly between 0 and 1, then for any event A,* $\boxed{P(A) = P(A|B)P(B) + P(A|B^C)P(B^C)}$.

4. Calculate $P(F = 6)$. Hint: Let B be the event of rolling a 6 and use the Law of Total Probability.

5. Calculate $P(F = 5)$

6. Calculate $P(F = 0)$

Challenge. Finish finding the PMF for F.

7. Suppose you know that your friend ran the die-coin experiment and flipped 5 heads ($F = 5$). Calculate the conditional probabilities of your friend having rolled 1, 2, 3, 4, 5, and 6 on the die. Which was most likely to have been her roll?

Challenge. Repeat the last problem, but suppose your friend got $F = 1$ instead of $F = 5$.