# CONTINUOUS RANDOM VARIABLES

A **continuous random variable** is a random variable which takes a continuum of possible values. Our main method of working with discrete random variables, the probability mass function, doesn't work for continuous random variables. Instead we have the **probability density function (PDF)**. The cumulative distribution function (CDF) is the same as ever: $F(x) = P(X \leq x)$.

**Definition.** Let $X$ be a continuous random variable. A **probability density function (PDF)** for $X$ is any function $f$ such that for any numbers $a$ and $b$ with $a < b$

$$P(a < X < b) = \int_a^b f(x)dx$$

**Theorem.** *A function $f$ may be the density function of a random variable if*

*(1)* $f(x) \geq 0$ *for all* $x$ *and*

*(2)* $\int_{-\infty}^{\infty} f(x)dx = 1.$

**1.** Suppose that a random variable $X$ has PDF $f(x) = kx^2$ if $0 < x < 1$ for some constant $k$.

a) Determine the value of the constant $k$.

b) Calculate $P(X \leq \frac{1}{2})$

c) Find the CDF of $X$.

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**2.** Find a PDF for the random variable with CDF $F(x) = \begin{cases} 1 - \frac{1}{x^2} & \text{if } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$

**3.** A random variable $Z$ with the standard normal distribution has PDF $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ for all $x$. This function doesn't have an elementary antiderivative, so the CDF cannot be described by an easy formula (like the CDF of problem 1 or 2). Some values of the CDF are given in the following table; use these to calculate the desired probabilities.

| $x$ | $-0.5$ | $0.0$ | $0.5$ | $0.8$ | $2.0$ |
|---|---|---|---|---|---|
| $F(x)$ | 0.3095 | 0.5 | 0.6915 | 0.7881 | 0.9972 |

a) $P(Z \leq 0.8)$

b) $P(Z < 0.0)$

c) $P(0.0 \leq Z \leq 0.8)$

d) $P(0 \geq Z)$

**Definition.** The **expected value** or **mean** of a continuous random variable with PDF $f$ is $\mu = \displaystyle\int_{-\infty}^{\infty} x f(x)dx$.

The **median** is the number $\tilde{\mu}$ such that $\displaystyle\int_{-\infty}^{\tilde{\mu}} f(x)dx = 0.5$.

**4.** Find the mean and median of a random variable with PDF $f(x) = e^{-x}$ if $x > 0$.