

**Exercise 1.** The joint probability mass function for two discrete random variables  $X$  and  $Y$  is given in the table below.

| p(x,y) |   | Y     |      |       |
|--------|---|-------|------|-------|
|        |   | 0     | 2    | 4     |
| X      | 0 | 0.1   | 0.15 | 0.025 |
|        | 1 | 0.025 | 0.25 | 0.075 |
|        | 5 | 0.075 | 0.1  | 0.2   |

- Find the marginal probability mass functions  $p_X(x)$  and  $p_Y(y)$ .
- Use the marginal pmfs of part (a) to calculate  $\mu_X$ ,  $\mu_Y$ ,  $\sigma_X$  and  $\sigma_Y$ .
- Calculate  $Cov(X, Y)$  using the shortcut formula  $Cov(X, Y) = E(XY) - \mu_X\mu_Y$ .
- Calculate the correlation coefficient of  $X$  and  $Y$ .
- Calculate  $P(X > Y)$  and  $P(X = Y)$ .
- Calculate the expected value of the average of  $X$  and  $Y$ ,  $E\left(\frac{X+Y}{2}\right)$ .