

Exercise 1. The joint probability mass function for two discrete random variables X and Y is given in the table below.

p(x,y)		Y		
		0	2	4
X	0	0.1	0.15	0.025
	1	0.025	0.25	0.075
	5	0.075	0.1	0.2

- a) Find the marginal probability mass functions $p_X(x)$ and $p_Y(y)$.

$$p_X(x) = \begin{cases} 0.275 & x = 0 \\ 0.35 & x = 1 \\ 0.375 & x = 5 \end{cases}$$

$$p_Y(y) = \begin{cases} 0.2 & y = 0 \\ 0.5 & y = 2 \\ 0.3 & y = 4 \end{cases}$$

- b) Use the marginal pmfs of part (a) to calculate μ_X , μ_Y , σ_X and σ_Y .

$$\mu_X = 2.225 \text{ and } \sigma_X \approx 2.185$$

$$\mu_Y = 2.2 \text{ and } \sigma_Y = 1.4$$

- c) Calculate $Cov(X, Y)$ using the shortcut formula $Cov(X, Y) = E(XY) - \mu_X\mu_Y$.

$$Cov(X, Y) = 0.905$$

- d) Calculate the correlation coefficient of X and Y .

$$\rho_{X,Y} \approx 0.2958$$

- e) Calculate $P(X > Y)$ and $P(X = Y)$.

$$P(X > Y) = 0.4 \text{ and } P(X = Y) = 0.1$$

- f) Calculate the expected value of the average of X and Y , $E\left(\frac{X+Y}{2}\right)$.

$$E\left(\frac{X+Y}{2}\right) = 2.2125$$