

In this experiment we roll a pair of fair dice, one red and one green. We can think of the sample space as consisting of ordered pairs:  $(1, 1)$  represents a roll of 1 on both dice;  $(1, 2)$  would represent a roll of 1 on the red die and 2 on the green die;  $(2, 1)$  would be 2 on the red die and 1 on the green die.

1. What is the total number of possible rolls?

Of course the thing we're really interested in when rolling a pair of dice is the sum of the numbers shown on the dice. Let  $X$  be this sum. Then  $X$  is called a *random variable* because it is a number (i.e.  $2, 3, 4, \dots, 12$ ) and its exact value depends on the outcome of a random process. For example, the outcomes  $(1, 2)$  and  $(2, 1)$  give  $X = 3$ . And these are the only outcomes that give  $X = 3$ . Thus it makes sense to use  $X = 3$  as a way of denoting the event  $\{(1, 2), (2, 1)\}$ . Combining this with the answer from question 1 allows us to calculate  $P(X = 3)$ , the probability that  $X = 3$ . Similarly,  $X = 4, X = 5$ , etc. each correspond to an event.

2. Finish filling out the table.

$X$	Probability	Number of rolls	Rolls
2		1	$(1, 1)$
3		2	$(1, 2)$ and $(2, 1)$
4			
5			
6			
7			
8			
9			
10			
11			
12			

3. Let  $A$  be the event that you roll a 3 on the red die. Let  $B$  be the event that you roll an even number on the green die.

a. Calculate  $P(A)$  and  $P(B)$ .

b. Are  $A$  and  $B$  independent events? Why or why not? Calculate  $P(A \cap B)$  and  $P(A \cup B)$ .

c. Calculate  $P(X = 5|A)$  and  $P(A|X = 5)$ . Are  $A$  and  $X = 5$  independent events?

d. Calculate  $P(X = 7|A)$  and  $P(A|X = 7)$ . Are  $A$  and  $X = 7$  independent events?

e. Calculate  $P(X = 5|B)$  and  $P(B|X = 5)$ . Are these independent events?

e. Calculate  $P(X = 7|B)$  and  $P(B|X = 7)$ . Are these independent events?

e. Find a number  $n$  such that the event  $X = n$  is *dependent* on the event  $B$ .