Thoerem 1 (The Central Limit Theorem)**.** *Let $X_1, X_2, \ldots X_n$ be iid with $E(X_i) = \mu$ and $V(X_i) = \sigma^2$. If n is sufficiently large then $\overline{X}$ has an approximately normal distribution with mean $\mu_{\overline{X}} = \mu$ and variance $V(\overline{X}) = \frac{\sigma^2}{n}$.*

Exercise 1 (Exercise 52)**.** The lifetime of a certain type of battery is normally distributed with a mean value of 10 hours and a standard deviation of 1 hour. There are four batteries in a package. 95% of all packages will have a total lifetime less than what value?

Exercise 2. Suppose that M&Ms are produced so that the average weight of an M&M is 2.2 g with a standard deviation of 0.4 g.

a) Use the central limit theorem to estimate the probability that the total weight of a package of 36 M&Ms is less than 75 g.

b) Estimate the probability that the total weight of a package of 100 M&Ms exceeds 225 g.

c) Find the value c such that about 98% of all packages of 100 M&Ms will have a total weight between $220 - c$ and $220 + c$.

Note: in the real world M&Ms are sold by weight and not by quantity.

In real world applications of statistics we usually wish to use data from a sample to make inferences about population parameters. For example, the sample mean (for a random sample) is a reasonable estimate for the population mean, so we say that $\overline{X}$ is a *point estimator* for the parameter μ. A point estimator $\hat{\Theta}$ for a parameter Θ is *unbiased* if $E(\hat{\Theta}) = \Theta$. Proposition 1 of section 5.4 tells us that $\overline{X}$ is an unbiased estimator for μ.

The *standard error* of an estimator $\hat{\Theta}$ is its standard deviation $\sigma_{\hat{\Theta}}$. The standard error is a measurement of the accuracy of the estimator. Unfortunately the calculation of $\sigma_{\hat{\Theta}}$ often involves unknown parameters. The solution is to use an estimator for the unknown parameters. This gives the *estimated standard error* of the estimator $\hat{\Theta}$.

For example, the central limit theorem tell us that for large n the standard error for $\overline{X}$ is $\frac{\sigma}{\sqrt{n}}$ (where σ is the population standard deviation). An unbiased estimator for σ is the sample standard deviation S given by

$$S^2 = \frac{\sum(X_i - \overline{X})^2}{n-1}.$$

The estimated standard error of $\overline{X}$ is then

$$\frac{S}{\sqrt{n}}.$$

Exercise 3. Tests of the shear strength of 10 random spot welds yield the following data (psi):

392	376	401	367	389	362	409	415	358	375

Assume that we know shear strength to be normally distributed.

a) Estimate the average shear strength of a spot weld.

b) Calculate the estimated standard error of your estimate in part a.