

The exam will cover chapter 2 and chapter 3 up to (and including) section 3.5. Calculators are allowed but will not be required. The following equations will be provided along with the exam (it will be up to you to know what they mean):

1. $V(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$.
2. The pmf of binomial distributions:

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}.$$

3. Appropriate tables of the cdf for the binomial distribution;
4. The expected value and variance of an rv $X \sim \text{Bin}(n, p)$: $E(X) = np$ and $V(X) = np(1-p)$.
5. The pmf of a hypergeometric distribution:

$$h(x; n, M, N) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}.$$

6. The expected value and variance of a hypergeometric rv X with pmf $h(x; n, M, N)$:

$$E(X) = n \left(\frac{M}{N} \right) \text{ and } V(X) = \left(\frac{N-n}{N-1} \right) n \left(\frac{M}{N} \right) \left(1 - \frac{M}{N} \right).$$

7. The pmf of a negative binomial distribution:

$$nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x.$$

8. The expected value and variance of a negative binomial rv X with pmf $nb(x; r, p)$:

$$E(X) = \frac{r(1-p)}{p} \text{ and } V(X) = \frac{r(1-p)}{p^2}.$$

Some of the things you should know by heart:

1. The sample space \mathcal{S} is the collection of all possible outcomes of an experiment.
2. An event is a subset of the sample space.
3. If all outcomes are equally likely, then the probability of an event A is

$$\frac{\text{number of outcomes in } A}{\text{number of outcomes in } \mathcal{S}}.$$

4. Basic facts about probabilities:

- (a) $0 \leq P(A) \leq 1$;
- (b) $P(\mathcal{S}) = 1$ and $P(\emptyset) = 0$;

(c) If A_1, A_2, A_3 are mutually disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$

(d) $P(A) + P(A') = 1$;

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

5. Counting techniques:

(a) Product rule for counting;

(b) $P_{k,n} = \frac{n!}{(n-k)!}$;

(c) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

6. Conditional probability:

(a) If $P(B) \neq 0$, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)};$$

(b) $P(A \cap B) = P(A|B)P(B)$;

(c) The Law of Total Probability: if A_1, A_2, \dots, A_n are mutually exclusive and exhaustive events, then

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots + P(B|A_n)P(A_n);$$

(d) Bayes' Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

7. Independence. The following are all equivalent:

(a) Events A and B are independent;

(b) $P(A|B) = P(A)$;

(c) $P(A \cap B) = P(A)P(B)$;

(d) A and B' are independent.

8. Random Variables:

(a) pmf and cdf;

(b) Expected value $E(X) = \mu = \sum xp(x)$;

(c) Rules for expected value and variance:

$$E(aX + b) = aE(X) + b \text{ and } V(aX + b) = a^2V(X);$$

(d) Standard deviation: $\sigma = \sqrt{V(X)}$.