Historical data shows that about 120 meteorites fall to Earth every 20 years. Define a random variable X_t to be the number of meteorites that fall in the next t years.

Exercise 1. Explain what $\mathrm{E}(X_t)$ is and figure out what value it should be. You may want to consider X_1 (the number of meteorites in one year) first.

$\mathrm{E}(X_t)$ is the expected number of meteorites in t years.

$$\mathrm{E}(X_1) = \frac{120}{20} = 6$$
$$\mathrm{E}(X_t) = 6t$$

For any fixed t the rv X_t has a *Poisson distribution* with parameter λ, where λ is the expected number of meteorites in t years. The pmf of X_t is

$$p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}.$$

Exercise 2. Express $p(x;\lambda)$ as function of x and t (i.e. plug the answer from exercise 1 into the formula for $p(x;\lambda)$).

$$\lambda = \mathrm{E}(X_t) = 6t$$
$$p(x;6t) = \frac{e^{-6t}(6t)^x}{x!}$$

Exercise 3. Calculate the following.

a) $\mathrm{P}(X_{0.5} = 3)$.

$$\mathrm{P}(X_{0.5} = 3) = p(3;3) = \frac{e^{-3}3^3}{3!} \approx 0.224$$

b) $\mathrm{P}(X_1 = 6)$

$$\mathrm{P}(X_1 = 6) = p(6;6) = \frac{e^{-6}6^6}{6!} \approx 0.1606$$

c) $\mathrm{P}(X_2 = 12)$

$$\mathrm{P}(X_2 = 12) = p(12;12) = \frac{e^{-12}(12)^12}{12!} \approx 0.1144$$

Exercise 4. Use the tables of Cumulative Poisson Probabilities in the book to find the following.

a) $\mathrm{P}(X_{0.5} \le 3)$.
$\lambda = 3$ and $x = 3$. The table gives 0.647.

b) $\mathrm{P}(X_1 \le 6)$.
$\lambda = 6$ and $x = 6$. The table gives 0.606.

c) $\mathrm{P}(X_{1.5} \le 9)$.
$\lambda = 9$ and $x = 9$. The table gives 0.587.

d) $\mathrm{P}(X_{2.5} \leq 15)$.

$\lambda = 15$ and $x = 15$. The table gives 0.568.

e) Speculate about what value these are trending toward and explain what this means.

I happen to know that as λ increases the Poisson distribution gets closer to the normal distribution. This means that $\mathrm{P}(X_t \leq 6t) \to \frac{1}{2}$ as $t \to \infty$. In other words, over long time periods the chance of having more than the expected number of meteorites is equal to the chance of having less than the expected number.

Poisson random variables are used to model many situations in which something occurs at random but at a known frequency (over time or space). For example, the number of customers at a store over a day, or the number of people processed by the DMV in an hour (relevant if you're at the end of a long line), or the number of typos in a page of a book, or the number of trees of a certain species in a square mile, or the number of mutations in mitochondrial DNA over a fixed time, etc.

Poisson distributions can also be used to approximate binomial distributions when the number of things selected is large and the probability of success is small (rule of thumb: $n > 50$ and $np < 5$). For example, about 0.5% of women are color blind. Let Y be the number of color blind women in a random sample of 800 women. Then $Y \sim \mathrm{Bin}(800, 0.005)$. Actually working with $b(x; 800, 0.005)$ is cumbersome and the tables in the back of the book don't include such large values for n. Fortunately Y is approximately Poisson with parameter $\lambda = 800(0.005) = 4$ (and this approximation is very good).

Exercise 5. Use the Poisson distribution to find approximate solutions to the following.

a) $\mathrm{P}(Y = 0)$.

$$p(0; 4) = \frac{e^{-4}(4)^0}{0!} = e^{-4} \approx 0.0183$$

Note: by convention $0! = 1$.

b) $\mathrm{P}(Y \geq 6)$

Use the table in the book with $\lambda = 4$.

$$\mathrm{P}(Y \geq 6) = 1 - \mathrm{P}(Y \leq 5) = 1 - 0.785 = 0.215$$