FORMULAS

$E(X) = \sum_x xf(x)$ $\quad Var(X) = E(X^2) - [E(X)]^2$ $\quad$ Bayes' Rule: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

The Law of Total Probability: $P(B) = P(B|A)P(A) + P(B|A')P(A')$.

DISTRIBUTIONS

Bernoulli distribution with parameter θ: pdf $f(x) = \begin{cases} \theta & \text{if } x = 1 \\ 1-\theta & \text{if } x = 0 \end{cases}$, mean $\mu = \theta$, variance $\sigma^2 = \theta(1-\theta)$.

Geometric distribution with parameter θ: pdf $f(x) = \theta(1-\theta)^{x-1}$ for $x = 1, 2, 3, \ldots$, mean $\mu = \frac{1}{\theta}$, variance $\sigma^2 = \frac{1-\theta}{\theta^2}$.

Binomial distribution with parameters θ and n: pdf $f(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ for $x = 0, 1, 2, \ldots, n$, mean $\mu = n\theta$, variance $\sigma^2 = n\theta(1-\theta)$.

Poisson distribution with parameter $\lambda > 0$: $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$, mean $\mu = \lambda$, and variance $\sigma^2 = \lambda$.

Uniform continuous distribution on the interval (a, b): pdf $f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$, mean $\mu = \frac{a+b}{2}$, and variance $\sigma^2 = \frac{(b-a)^2}{12}$.

Exponential distribution with parameter $\lambda > 0$: pdf $g(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$, cdf $G(x) = \begin{cases} 1 - e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$, mean $\mu = \frac{1}{\lambda}$, and variance $\sigma^2 = \frac{1}{\lambda^2}$.

Normal distribution with parameters μ and $\sigma > 0$: pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, mean μ, and variance σ^2.

CONFIDENCE INTERVALS

- $100(1-\alpha)\%$ **confidence interval for** μ: $\overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$.
- $100(1-\alpha)\%$ **confidence interval for** μ: $\overline{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$.
- **Approximate** $100(1-\alpha)\%$ **confidence interval for** θ: $\hat{\theta} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$.
- $100(1-\alpha)\%$ **confidence interval for** σ^2: $\left(\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}\right)$.

TEST STATISTICS

- For **tests about the mean** ($H_0 : \mu = \mu_0$):

  $z = \frac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ for samples from a population with a known variance σ^2 (all sample sizes if the population is normal, otherwise just for large samples);

  $z = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ for large samples;

  $t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ for small samples ($n \le 30$ or as big as your t-table goes) from a normally distributed population. The test statistic has a t distribution with $n-1$ degrees of freedom.

- For **tests about a population proportion** ($H_0 : \theta = \theta_0$):

  x (X is binomial with parameters n and θ_0, works best for small samples);

  $z = \frac{\hat{\theta} - \theta_0}{\sqrt{\frac{\theta_0(1-\theta_0)}{n}}} = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}$ for large samples (both $n\theta_0 \ge 10$ and $n(1-\theta_0) \ge 10$).

- For **tests about the variance** ($H_0 : \sigma^2 = \sigma_0^2$):

  $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ for samples from a normally distributed population. The test statistic has a chi-square distribution with $n-1$ degrees of freedom.

- For **tests about the standard deviation**: square the standard deviations and do a test for the variance.
- For **tests about the difference of two means** ($H_0 : \mu_1 - \mu_2 = \delta_0$) the test statistics are:

  $z = \dfrac{\overline{x}_1 - \overline{x}_2 - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ for samples from populations with a known variances (all sample sizes if the populations are normal, otherwise just for large samples);

  $z = \dfrac{\overline{x}_1 - \overline{x}_2 - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ for large samples;

  $t = \dfrac{\overline{x}_1 - \overline{x}_2 - \delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ for small samples from normally distributed populations with the same variance ($n_1 + n_2 - 2$ df).

  Recall $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$.