Exam 1 will cover all of chapters 1 and 2 and some of chapter 3 (through section 3.4 plus the geometric distribution in section 3.5). Some (but not all) of the things you should know:

The *sample space* $\mathcal{S}$ is the collection of all possible outcomes of an experiment. An *event* is a subset of the sample space.

Counting:

- If all the outcomes are equally likely, then the probability of an event A is $\dfrac{\text{number of outcomes in } A}{\text{number of outcomes in } \mathcal{S}}$
- Product rule for counting
- Combinations: $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$
- Permutations: $P_{k,n} = \dfrac{n!}{(n-k)!}$

The number of ways to select k elements from an n-element set is...

	Order matters	Order doesn't matter
With replacement	n^k	?
Without replacement	$\dfrac{n!}{(n-k)!}$	$\binom{n}{k}$

Basic facts about probabilities:

- $0 \leq P(A) \leq 1$
- $P(\mathcal{S}) = 1$ and $P(\emptyset) = 0$
- If $A_1, A_2, A_3, \ldots$ are mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$$

- $P(A) + P(A') = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional probability:

- The definition: if $P(B) \neq 0$, then $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- Product version: $P(A \cap B) = P(A|B)P(B)$
- Bayes' Rule: $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$
- The Law of Total Probability: if $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events, then

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_n)P(A_n)$$

Events A and B are independent if and only if:

- $P(A \cap B) = P(A)P(B)$ or
- $P(A|B) = P(A)$ or
- $P(B|A) = P(B)$.

Random Variables:

- Probability distribution function (pdf) $f(x) = P(X = x)$

- Cumulative distribution function (cdf) $F(x) = P(X \leq x)$
- Expected value $\mu = E(X) = \sum_x x f(x)$
- Expected value of a function $h(x)$: $E[h(X)] = \sum_x h(x) f(x)$
- Variance $\sigma^2 = Var(X) = E\left[(X-\mu)^2\right] = E(X^2) - [E(X)]^2$
- $E(aX+b) = aE(X)+b$
- Standard deviation: $\sigma = \sqrt{\sigma^2}$.

The **product rule** (or **multiplication rule**) says that when dealing with a sequence of things you should multiply. In contrast, when you're dealing with an experiment in which one thing *or* another may happen, you should add (and perhaps subtract the intersection). The same applies to both counting and calculations of probabilities.

Example. Standard 5-card poker hands are ranked so that the best kind of hand is the least likely to occur. The total number of 5-card poker hands is $\binom{52}{5} = 2598960$. Each hand is equally likely, so the probability of any particular kind of hand (e.g. full house) is simply the number of hands of that kind divided by 2598960.

- The number of royal flushes is 4.
- The number of four of a kinds is $\binom{13}{1}\binom{48}{1} = 13(48) = 624$.
- The number of full houses is $\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{3} = 13(6)(12)(4) = 3744$.

Example. The die-coin experiment involved rolling a die *then* flipping a coin the number of times shown on the die. This is a sequence, which means we need to multiply. The probability of rolling a 1 and then flipping tails is $\frac{1}{6}\left(\frac{1}{2}\right)$. The probability of rolling a 2, then flipping tails, and then flipping tails again is $\frac{1}{6}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$. And so on. Calculating the probability of flipping 0 heads in this experiment involves addition. If we let F be the number of heads flipped, then we're calculating $P(F=0)$. We get $F=0$ by rolling a 1 and then flipping tails, *or* by rolling a 2 and then flipping tails twice, *or* by rolling a 3 and then flipping tails 3 times, ..., *or* by rolling a 6 and then flipping tails 6 times. Thus

$$P(F=0) = \frac{1}{6}\left(\frac{1}{2}\right) + \frac{1}{6}\left(\frac{1}{2}\right)^2 + \frac{1}{6}\left(\frac{1}{2}\right)^3 + \frac{1}{6}\left(\frac{1}{2}\right)^4 + \frac{1}{6}\left(\frac{1}{2}\right)^5 + \frac{1}{6}\left(\frac{1}{2}\right)^6.$$

A little number crunching shows that $P(F=0) = \frac{21}{128}$. We can also use this to calculate the probability of flipping at least one head:

$$P(F \geq 1) = 1 - P(F=0) = \frac{107}{128}.$$

Example. How many ways are there to order the letters MISS? First lets consider 4 distinct letters $\mathrm{MIS_1S_2}$. These can be ordered in $4! = 24$ ways. These 24 orderings come in pairs that differ only in the ordering of $\mathrm{S_1}$ and $\mathrm{S_2}$. One such pair is $\mathrm{MS_1IS_2}$ and $\mathrm{MS_2IS_1}$, for example. If we drop the subscripts, then both members of the pair look the same: each gives the same ordering of the letters MISS. Therefore the number of orderings of MISS is $\frac{4!}{2} = 12$.

Example. Let X be the value rolled on a die. The expected value of X is

$$\begin{aligned} E(X) &= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) \\ &= \frac{1}{6}(1+2+3+4+5+6) \\ &= \frac{7}{2} \end{aligned}$$

Note that $E(X)$ need not be a possible value of X.

So what formulas will I give you? I will give you pdfs, means, and variances for our named distributions (Bernoulli, binomial, and geometric). I will also include the formulas $E(X) = \sum_x x f(x)$, $Var(X) = E(X^2) - [E(X)]^2$, and for Bayes' rule, though I do expect you to have these memorized soon. If you spot other formulas you'd like me to include, just email me a request.