$$\boxed{\binom{n}{k} = \frac{n!}{k!(n-k)!}}$$

Example 1. For $n \geq 2$ we find $\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)(n-2)(n-3)\dots(2)1}{2(1)[(n-2)(n-3)\dots(2)1]} = \frac{n(n-1)}{2}$.

1. Famiarize yourself with binomials by calculating the following (some answers will be numbers, others will depend on n).

a) $\binom{n}{0}$

b) $\binom{n}{1}$

c) $\binom{n}{n-2}$

d) $\binom{n}{n-1}$

e) $\binom{n}{n}$

Example 2. This example and the following problem deal with 5-card poker hands dealt from a well-shuffled standard deck of 52 cards. We are interested in calculating the probability of being dealt different hands, which we can accomplish by counting. A full house consists of a pair and a three-of-a-kind. To count the number of full houses: first select 2 of the 13 kinds of card (counting order and without replacement this can be done in 13(12) ways), then select 2 of the first kind and 3 of the second (not counting order and without replacement this can be done in $\binom{4}{2}\binom{4}{3}$ ways). This gives a total of

$$13(12)\binom{4}{2}\binom{4}{3} = 13(12)(6)(4) = 3744.$$

2. We continue to think about 5-card poker hands.

a) How many total hands are there?

b) What is the probability of being dealt a full house?

c) What is the probability of being dealt four of a kind?

d) What is the probability of being dealt a flush (including royal and straight flushes)?

e) What is the probability of being dealt at least one heart?

3. Find the mistake in the following calculation of the number of 5-card hands that contain at least one heart:

> Order doesn't matter for the cards, so we can start by selecting any one of the 13 hearts. We can do this in 13 ways. We then add any 4 other cards from the remaining 51 to the hand. We can do this in $\binom{51}{4}$ ways. Therefore the total number of hands containing at least one heart is
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> $$13\binom{51}{4} = 3,248,700.$$