Names:
Math 321 | The die coin experiment | January 29, 2016

The die-coin experiment consists of rolling a fair die and then flipping a fair coin the number of times shown on the die. A sample space for this experiment is

$$S = \{(1, H), (1, T), (2, HH), (2, HT), (2, TH), (2, TT), \ldots, (6, TTTTTT)\}$$

but what we're really interested in is the numbers showing up in the experiment. First there's the number Rolled on the die, which we'll call R. Then there's the number of times we Flip heads, which we'll call F. The variables R and F are called *random variables* because their values will be determined by a random process (in this case the die-coin experiment).

1. What are the possible values for R? What is the probability that R takes each of these values?

2. Calculating the probability of F taking each of its possible values is difficult. Conditional probabilities turn out to be easier: they can be determined just by analyzing the experiment (not using the definition of conditional probability).

a) What are the possible values for F?

b) Determine the conditional probabilities that $F = 1$ given that $R = 1$, $R = 2$, $R = 3$, $R = 4$, $R = 5$, and $R = 6$.

Thoerem 1 (Law of Total Probability)**.** *If $A_1, A_2, \ldots, A_n$ are mutually exclusive events, then*

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_n).$$

The Law of Total Probability is often used together with the formula $P(B \cap A_i) = P(B|A_i)P(A_i)$.

3. Calculate P(F=1).

4. Suppose you know that your friend ran the die-coin experiment and flipped heads once ($F = 1$). Calculate the conditional probabilities of her having rolled 1, 2, 3, 4, 5, or 6 on the die. Which was most likely to have been her roll?