Names:

Math 321 | A pair of dice | February 8, 2016

This experiment consists of rolling a pair of fair dice, one red and one green. We can think of the sample space as a set of ordered pairs: $(1,1)$ is a roll of 1 on both dice, $(1,2)$ is a roll of 1 on the red die and 2 on the green die, etc. It may be helpful to keep track these outcomes in a table (you should fill in the entries below). The outcomes in this sample space are all equally likely, but what we're really interested in is the sum of the numbers rolled. Let the random variable N be this quantity (so $N = 2$ corresponds to $\{(1,1)\}$, $N = 3$ corresponds to $\{(1,2),(2,1)\}$, etc.). We'll also keep track of R, the number rolled on the red die.

		Green					
		1	2	3	4	5	6
Red	1						
	2						
	3						
	4						
	5						
	6						

1. Determine the probability distribution function of N. There's not a good formula here–just make a table.

2. Are the events $R = 1$ and $N = 7$ independent?

3. What number is the center of the distribution of N?

4. What number is the center of the distribution of R?

The usual method of finding the center of a distribution is to calculate the *expected value* (or *mean value*) of the random variable:

$$E(X) = \sum_x xf(x)$$

where the sum is taken over all possible values of the random variable X.

5. Calculate the expected values $E(R)$ and $E(N)$. Are these the same central values you identified in problems 3 and 4?

6. The Washington Lottery's Match 4 game involves selecting 4 numbers between 1 and 24. Winnings and some details are online: `http://www.walottery.com/Games/Match4/Default.aspx`.

a) Use the probabilities on the Match 4 web page to determine if order matters and if the selection of winning numbers is done with or without replacement.

b) Calculate the expected value of a Match 4 ticket.

7. A game, which we'll call St. Petersburg, starts with \$1 in the pot. A fair coin is then flipped until the first heads appears, at which point you win the pot. Each time the coin comes up tails the pot is doubled (so you win \$1 if the first flip is heads, \$2 for tails then heads, \$4 for tails, tails, heads, and so on).

a) What is the expected value of the game? (This is called the *fair price* for the game).

b) How much would you pay to play? What is the probability of a profit if you pay that price?

8. Petrograd is a new version of St. Petersburg that is the same except that the game stops at 20 flips if heads has not appeared. If this happens, you win nothing.

a) What is the fair price for this game?

b) What is the probability of a profit if you pay the fair price?