NAMES:
MATH 321 — CONTINUOUS RANDOM VARIABLES — FEBRUARY 22, 2016

A *continuous random variable* is a random variable which takes a continuum of possible values. Our main method of working with discrete random variables, the probability distribution function, doesn't work for continuous random variables. Instead we have the *probability density function (pdf)*. The cumulative distribution function (cdf) is the same as ever ($F(x) = P(X \leq x)$ for any random variable X).

Definition. Let X be a continuous random variable. A **density function** for X is any function f such that

$$P(a < X < b) = \int_a^b f(x)dx$$

for any real numbers a and b with $a < b$.

Thoerem. *A function f may be the density function of a random variable if and only if*

1. $f(x) \geq 0$ *for all* x *and*
2. $\displaystyle\int_{-\infty}^{\infty} f(x)dx = 1.$

1. Suppose that a random variable X has pdf $f(x) = \begin{cases} kx^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

a) Determine the value of the constant k.

b) Calculate $P(X \leq \frac{1}{2})$

c) Calculate $P(X \leq \frac{3}{2})$

d) Calculate $P(X \leq \frac{3}{4})$

2. A random variable with a uniform distribution on the interval $(0, 1)$ has pdf $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ Find the cdf of the random variable.

3. Find a pdf for the random variable with cdf $F(x) = \begin{cases} 1 - \frac{1}{x^2} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$. Hint: $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$.

4. A random variable Z with the standard normal distribution has pdf $f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ for all x. This function doesn't have an elementary antiderivative, so the cdf is hard to calculate. Some values of the cdf are given in the following table; use these to calculate the desired probabilities.

x	-0.5	0.0	0.5	0.8	2.0
$F(x)$	0.3095	0.5	0.6915	0.7881	0.9972

a) $P(-0.5 < Z \leq 0.8)$

b) $P(-0.5 \leq Z < 0.8)$

c) $P(Z = 0.8)$

d) $P(0 < Z)$

Definition. The *expected value* (or *mean*) of a continuous random variable with pdf f is $\mu = \int_{-\infty}^{\infty} xf(x)dx$. The *median* is the number $\tilde{\mu}$ such that $0.5 = \int_{-\infty}^{\tilde{\mu}} f(x)dx$. In terms of the cdf F, $\tilde{\mu}$ is the number such that $F(\tilde{\mu}) = 0.5$ (equivalently, $\tilde{\mu} = F^{-1}(0.5)$).

5. Find the mean and median of a random variable with pdf $f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$