Names:
Math 321 — Earthquakes — February 26, 2016

Definition. A random variable has a *Poisson distribution* with parameter $\lambda > 0$ if its probability distribution function is $p(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$.

Proposition 1. *If X has a Poisson distribution with parameter $\lambda > 0$, then $E(X) = \lambda$ and $Var(X) = \lambda$.*

Definition. A random variable has an *exponential distribution* with parameter $\lambda > 0$ if its probability density function is

$$g(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Proposition 2. *If X is exponentially distributed with parameter λ, then $E(X) = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$.*

1. Earthquakes occur pretty much at random and there's never more than one at the same place and time. It turns out that this kind of behavior is nicely modeled using a Poisson distribution. The PNW region has an average of about 9.2 earthquakes of magnitude 4.0 or greater each year (1070 such earthquakes since 1900 according to earthquake.usgs.gov, and according to my rough estimate for what the PNW region means). Let N_t be the number of earthquakes in the PNW over t years. We'll assume that N_t has a Poisson distribution.

a) What parameter λ does N_1 have?

b) What parameter does N_t have?

c) What is the probability that there will be at least one earthquake of magnitude 4.0 or greater before the end of the school year (70 days)?

(Technically, N_t is a family of random variables known as a Poisson process).

2. Let T be the duration between earthquakes in the PNW, that is the time t at which N_t changes from 0 to 1. We'll think of T as a continuous random variable.

a) Fill in the blank: $T > t$ if and only if $N_t =$ _______

b) Find the cumulative distribution function for T.

c) Find a probability density function for T and identify the distribution.

d) How long do you have to wait between earthquakes on average?

e) How long do you have to wait until the probability of an earthquake occurring exceeds 0.5?

Challenge. Let T_2 be the time to the second earthquake. Find the probability density function of T_2 (use the same process as for the last problem).