NAMES:

MATH 321 ESTIMATORS MARCH 18, 2016

1. Suppose that we collect a random sample of size 9 from a population that is uniformly distributed on the interval $[0, \beta]$. This makes the probability density function of the population $f(x) = \frac{1}{\beta}$ for $0 < x < \beta$.

a) Find a constant a such that $a\overline{X}$ is an unbiased estimator of β.

b) Let Y be the largest number in the sample. Find the cumulative distribution function of Y.

c) Find a constant b such that bY is an unbiased estimator of β.

d) Calculate the variances of $a\overline{X}$ and bY and use these numbers to decide which of the two unbiased estimators is better.

2. Let Z be a standard normal random variable.

a) Find a number z such that $P(|Z| < z) = 0.95$.

b) Rearrange the inequality of part a to fill in the blanks in the following expression:

$$P(Z - \_\_\_\_\_\_ < 0 < Z + \_\_\_\_\_\_) = 0.95$$

3. Let $\overline{X}$ be the mean of a random sample of size $n = 9$ from a normally distributed population with mean μ (unknown) and standard deviation $\sigma = 12$. This means that X is normally distributed with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}} = 4$.

a) Using your work in problem 2 as a guide, fill in the blanks in the following expression:

$$P(\overline{X} - \underline{\qquad\qquad} < \mu < \overline{X} + \underline{\qquad\qquad}) = 0.95$$

b) Samples are taken and you find $\overline{x} = 50$. Substitute this value in for $\overline{X}$ in part a to find the *95% confidence interval* for the population mean μ.

c) What's wrong with the expression $P(42.16 < \mu < 57.84) = 0.95$?

d) Your 95% confidence interval is actually just the interval $(42.16, 57.84)$. What do these numbers mean?