Definition. A random variable X with a **uniform continuous distribution** with parameters α and β (with $\alpha < \beta$) has the following probability density function: $\boxed{f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{elsewhere} \end{cases}}$.

Proposition. *A uniform continuous distribution with parameters α and β has mean $\mu = \frac{\alpha+\beta}{2}$, and variance $\sigma^2 = \frac{(\beta-\alpha)^2}{12}$.*

Definition. A random variable with an **exponential distribution** with parameter $\lambda > 0$ has the following probability density function: $\boxed{g(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$

Proposition. *An exponential distribution with parameter λ has mean $\mu = \frac{1}{\lambda}$ and variance $\sigma^2 = \frac{1}{\lambda^2}$.*

R Implementation. The parameter λ is called the **rate**. For example, a rate of 2 events per minute corresponds to an average (mean) of 0.5 minutes between events.

- **R** code for the CDF: `> pexp(x, λ)`
- Example: if X is exponential with a mean of 2.5, then $P(1 < X \leq 3)$ can be calculated using

```
> pexp(3, 0.4) - pexp(1, 0.4)
```

Definition. A random variable with a **normal distribution** with parameters μ and $\sigma > 0$ has the following probability density function: $\boxed{n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for all } x \in \mathbb{R}}$.

Proposition. *A normal distribution with parameters μ and σ has mean $\mu = \mu$ and variance $\sigma^2 = \sigma^2$.*

R Implementation. If $X \sim \text{norm}(\mu, \sigma)$, then the CDF is `> pnorm(x, μ, σ)`.

Of these distributions, only the uniform continuous distribution and the exponential distribution allow one to easily compute probabilities by hand. Calculations of probabilities for the rest of the distributions generally rely on a table or computational device. For a normally distributed random variable X this usually means **standardizing** the random variable: $Z = \dfrac{X-\mu}{\sigma}$ has a **standard normal distribution** with mean 0 and variance 1.

Definition. The **gamma function** is defined as $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$ for $\alpha > 0$.

Proposition. *For any positive integer n, $\Gamma(n) = (n-1)!$*

Definition. A random variable X with a **gamma distribution** with parameters $\alpha > 0$ and $\beta > 0$ has the following PDF: $\boxed{g(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$.

Proposition. *A gamma distribution with parameters α and β has mean $\mu = \alpha\beta$, and variance $\sigma^2 = \alpha\beta^2$.*

Definition. A random variable with a **chi-square distribution** with parameter $\nu > 0$ has the following probability density function: $\boxed{f(x) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\frac{\nu-2}{2}} e^{-\frac{\nu}{2}} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}}$ (a gamma distribution with $\alpha = \frac{\nu}{2}$ and $\beta = 2$).

Proposition. *A chi-square distribution with parameter ν has mean $\mu = \nu$ and variance $\sigma^2 = 2\nu$.*

Definition. A random variable T with **Student's t distribution** with parameter $\nu > 0$ (called **degrees of freedom**) has the following PDF: $\boxed{g(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}}$

Proposition. *Student's t distribution with paramter ν has mean $\mu = 0$ and variance $\sigma^2 = \frac{\nu}{\nu-2}$ (if $\nu > 2$).*