EXAM 2 FORMULAS

Definition. Let A and B be events with $P(B) \neq 0$. The **conditional probability of A given B** is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Definition. Events A and B are **independent** if and only if $P(A \cap B) = P(A)P(B)$.

Definition. A **random variable** X assigns a number to each outcome in the sample space S.

(1) All random variables have a **cumulative distribution function (CDF)**: $F(x) = P(X \leq x)$.
(2) A discrete random variable has a **probability mass function (PMF)**: $m(x) = P(X = x)$.
(3) A continuous random variable has a **probability density function (PDF)** $f(x)$ such that for any numbers a and b (with $a \leq b$) $P(a \leq X \leq b) = \int_a^b f(x)dx$

Thoerem. *The CDF of a random variable X satisfies the following:*

(1) it is non-decreasing: if $a \leq b$, then $F(a) \leq F(b)$
(2) $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$

Thoerem. *The PMF of any discrete random variable X satisfies the following:*

(1) $0 \leq p(x) \leq 1$ for all x
(2) $\sum_x p(x) = 1$ (where the sum is over all possible values of X)

Thoerem. *A PDF for a random variable satisfies the following:*

(1) $f(x) \geq 0$ for all x
(2) $\int_{-\infty}^{\infty} f(x)dx = 1$

Definition. The **expected value** (or **mean**) of a random variable:

(1) If X is a discrete RV with PMF $m(x)$, then $E(X) = \sum_x xm(x)$.
(2) If X is a continuous RV with PDF $f(x)$, then $E(X) = \int_{-\infty}^{\infty} xf(x)dx$.

Definition. The **variance** of a random variable: $\text{Var}(X) = \sigma^2 = E\left[(X-\mu)^2\right] = E\left(X^2\right) - [E(X)]^2$. The **standard deviation** is $\sigma = \sqrt{\sigma^2}$.

Definition. The PMF of a random variable having a **binomial distribution** with parameters n and p is

$$\boxed{b(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n}$$

Proposition. *The mean of a binomial distribution is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$.*

Definition. The PMF of a random variable having a **geometric distribution** with parameter p is

$$\boxed{g(n) = p(1-p)^{n-1} \text{ for } n = 1, 2, 3, \ldots}$$

Proposition. *The mean of a geometric distribution is $\mu = \frac{1}{p}$ and the variance is $\sigma^2 = \frac{1-p}{p^2}$.*

Definition. The PMF of a random variable having a **Poisson distribution** with parameter λ is

$$\boxed{p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \ldots}$$

Proposition. *The mean and variance of a Poisson distribution are $\mu = \lambda$ and $\sigma^2 = \lambda$.*

Definition. A random variable X with a **uniform continuous distribution** with parameters α and β (with $\alpha < \beta$) has the PDF $\boxed{f(x) = \frac{1}{\beta - \alpha} \text{ for } \alpha < x < \beta}$.

Proposition. *A uniform continuous distribution with parameters α and β has mean $\mu = \frac{\alpha+\beta}{2}$, and variance $\sigma^2 = \frac{(\beta-\alpha)^2}{12}$.*

Definition. A random variable with an **exponential distribution** with parameter $\lambda > 0$ has the following PDF and CDF $\boxed{g(x) = \lambda e^{-\lambda x} \text{ for } x > 0}$ and $\boxed{G(x) = \begin{cases} 1 - e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}}$

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Proposition. *An exponential distribution with parameter* λ *has mean* $\mu = \frac{1}{\lambda}$ *and variance* $\sigma^2 = \frac{1}{\lambda^2}$.

Definition. A random variable with a **normal distribution** with parameters μ and $\sigma > 0$ has the PDF

$$\boxed{n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for all } x \in \mathbb{R}}$$

Proposition. *A normal distribution with parameters* μ *and* σ *has mean* $\mu = \mu$ *and variance* $\sigma^2 = \sigma^2$ *(and standard deviation* $\sigma = \sigma$*).*

Thoerem (Standardizing)**.** *If* $X \sim N(\mu, \sigma)$*, then* $Z = \dfrac{X - \mu}{\sigma}$ *is a standard normal random variable.*