CONDITIONAL PROBABILITIES

Theorem (The Law of Total Probability). $\boxed{P(A) = P(A|B)P(B) + P(A|B^C)P(B^C)}$

1. An October, 2019 Pew Center survey of US adults asked respondents if they "think the federal government is doing too little to reduce of the effects of climate change." They report the following results:

(1) Of those who preferred the Democratic Party, 90% said yes;
(2) Of those who preferred the Republican Party, 39% said yes;
(3) 67% of all respondents said yes.

What percent of respondents preferred the Democratic Party? (Assume that everyone had to choose either Democratic or Republican; use the Law of Total Probability).

The remaining problems concern the die-coin experiment, which consists of rolling a (fair, 6-sided) die and then flipping a fair coin the number of times shown on the die. For example, if you roll a 1 you'll flip the coin once, but if you roll a 2 you'll flip the coin twice. A sample space for this experiment is

$$S = \{(1, H), (1, T), (2, HH), (2, HT), (2, TH), (2, TT), \ldots, (6, TTTTTT)\}.$$

Note that the outcomes are not equally likely.

We'll deal with two random variables in this experiment:

- R, the number you roll on the die;
- F, the number of times you flip heads.

Our goal is to find the **probability mass function** (pmf) for F.

Example. We start with the pmf for R: this random variable has possible values $1, 2, 3, 4, 5, 6$ and it takes each of those values with probability $1/6$. The pmf for R is thus $m(x) = 1/6$ for $x = 1, 2, 3, 4, 5, 6$.

2. What are the possible values for F? (Do not attempt to calculate probabilities yet).

Calculating the probabilities for F is difficult unless we are given information about the roll of the die. For example, if we know that $R = 1$, then we know that $F = 0$ with probability $1/2$. This is a **conditional probability** and is expressed symbolically as $P(F = 0|R = 1) = 1/2$ (read as "the probability of $F = 0$ given $R = 1$").

3. What is the probability of $F = 1$ given $R = 1$?

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4. Suppose that you have rolled a 2 (and are about to flip the coin twice). Calculate the following conditional probabilities:

a) $P(F = 0|R = 2)$

b) $P(F = 1|R = 2)$

c) $P(F = 2|R = 2)$

5. Calculate $P(F = 6)$. Hint: Let B be the event of rolling a 6 and use the Law of Total Probability.

6. Calculate $P(F = 5)$

7. Calculate $P(F = 0)$

Challenge. Finish finding the pmf for F.

8. Suppose you know that your friend ran the die-coin experiment and flipped 5 heads ($F = 5$). Calculate the conditional probabilities of your friend having rolled 1, 2, 3, 4, 5, and 6 on the die. Which was most likely to have been her roll?

Challenge. Repeat the last problem, but suppose your friend got $F = 1$ instead of $F = 5$.