EARTHQUAKES

Definition. A random variable has a *Poisson distribution* with parameter $\lambda > 0$ if its probability distribution function is $p(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$.

Proposition 1. *If X has a Poisson distribution with parameter $\lambda > 0$, then $E(X) = \lambda$.*

1. Earthquakes occur pretty much at random and there's never more than one at the same place and time. It turns out that this kind of behavior is nicely modeled using a Poisson distribution. The PNW region has an average of about 9.2 significant earthquakes (magnitude 4.0 or greater) each year (1070 such earthquakes since 1900 according to earthquake.usgs.gov, and according to my rough estimate for what the PNW region means). Let N_t be the number of significant earthquakes in the PNW over t years. N_t should have a Poisson distribution with a parameter that depends on t. For example N_1 is the number of significant earthquakes in one year, so it should have a Poisson distribution with $\lambda = 9.2$.

a) What is the value of λ for N_2?

b) What is the value of λ for N_t (as a function of t)?

c) What is the probability that there will be at least one significant earthquake before the end of the school year (about 70 days)?

(Technically, N_t is a family of random variables known as a **Poisson process**).

2. Let T be the time to the next significant earthquakes in the PNW. This means T is a random variable that is connected to the random variable N_t. Note that T as a continuous random variable.

a) Fill in the blank: $T > t$ if and only if $N_t =$ ______

b) Use your last answer to find the CDF for T:

$$\begin{aligned} F(t) &= P(T \leq t) \\ &= 1 - P(T > t) \\ &= 1 - P\Big(N_t = \underline{\qquad}\Big) \\ &= \underline{\qquad\qquad\qquad} \end{aligned}$$

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c) Differentiate to find a probability density function for T.

d) What is the mean wait time to the next earthquake?

e) How long until the probability of an earthquake occurring exceeds 0.5? (Use the CDF for T).

Challenge. Let T_2 be the time to the second earthquake. Find the probability density function of T_2 (use the same process as for the last problem).