321 Functions of Random Variables (Solutions) 3/12/20

1. $E\left(\frac{X-50}{4}\right) = \frac{1}{4}E(X-50) = \frac{1}{4}\left[E(X)-50\right] = \frac{1}{4}[50-50] = \boxed{0}$

$Var\left(\frac{X-50}{4}\right) = E\left[\left(\frac{X-50}{4}-0\right)^2\right]$ def. of variance

$= E\left[\frac{1}{16}(X^2-100X+2500)\right]$

$= \frac{1}{16}\left[E(X^2)-100E(X)+2500\right]$

$= \frac{1}{16}\left[E(X^2)-\underbrace{2500}_{[E(X)]^2}\right]$

$= \frac{1}{16}Var(X)$ using the def. of variance

$= \frac{1}{16}(4^2) = \boxed{1}$

2. $E(\bar{X}) = E\left[\frac{1}{n}\sum_{i=1}^{n}X_i\right] = \frac{1}{n}\sum_{i=1}^{n}E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\mu = \frac{1}{n}(n\mu) = \mu.$

$Var(\bar{X}) = Var\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}Var(X_i) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{1}{n^2}n\sigma^2 = \frac{\sigma^2}{n}.$

theorem 4

theorem 2

3. a) Mean mass for video spokes: $36(6.7) = 241.2$ g

Mean mass for these spokes: $32(7.5) = 240$ g

These spokes are lighter on average.

3 b) $E(T) = 240$ $Var(T) = 32(0.5^2) = 8$

$$P(T > 245) = P\left(\frac{T-240}{\sqrt{8}} > \frac{245-240}{\sqrt{8}}\right)$$

$$\approx P(Z > 1.7678)$$

$$\approx 0.0385$$

These spokes are less likely to total more than 245g.

Note that increasing the standard deviation to 0.6g for these spokes results in $P(T>245) \approx P(Z>1.4731) \approx 0.07$, which would make these spokes more likely to exceed 245g.

4. Single roll of the die: $\mu = \frac{1}{6}(1+2+3+4+5+6) = \frac{7}{2}$

$$\sigma^2 = \frac{1}{6}(1+4+9+16+25+36) - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

T: sample total, $n = 64$

$$P(T > 200) \approx P\left(Z > \frac{200 - 64(7/2)}{\sqrt{64(35/12)}}\right)$$

$$\approx P\left(Z > \frac{-24}{13.6626}\right)$$

$$\approx 0.9605$$

5. $\bar{X} \sim \text{norm}\left(100, \frac{10}{\sqrt{n}}\right)$

$$P(98 < \bar{X} < 102) = P\left(\frac{-2}{10/\sqrt{n}} < Z < \frac{2}{10/\sqrt{n}}\right)$$

$$= P(-0.2\sqrt{n} < Z < 0.2\sqrt{n})$$

$$= 1 - 2\,\text{pnorm}(-0.2\sqrt{n}) \quad \text{using symmetry}$$

a) 0.1585

b) 0.6827

c) 0.9545

d) 1

6. a) $T \sim \text{binom}(64, 1/6)$

b) $P(T \geq 12) = 1 - \text{pbinom}(11, 64, 1/6) \approx 0.3769$

c) $P(T \geq 12) \approx P\left(Z \geq \frac{12 - 64(1/6)}{\sqrt{64(1/6)(5/6)}}\right) \approx 1 - \text{pnorm}(0.4472)$

≈ 0.3274

A better approximation can be achieved using a correction for continuity:

$$P(T \geq 12) \approx P\left(Z \geq \frac{11.5 - 64(1/6)}{\sqrt{64(1/6)(5/6)}}\right) \approx 1 - \text{pnorm}(0.2795)$$

≈ 0.3899

But why mess with this when you can get an exact answer using a binomial distribution?