Solutions to the worksheet on point estimation (3/16).

1. a) PDF for $\hat{X}$: $f(x) = \begin{cases} \frac{10}{\beta^{10}} x^9 & \text{for } 0 < x < \beta \\ 0 & \text{elsewhere} \end{cases}$

b) $E(\hat{X}) = \int_0^\beta \left(\frac{10}{\beta^{10}} x^9\right) x \, dx$

$= \int_0^\beta \frac{10}{\beta^{10}} x^{10} \, dx$

$= \frac{10}{\beta^{10}} \left(\frac{1}{11} x^{11}\right) \Big|_0^\beta$

$= \frac{10}{11} \beta$

c) $E(c\hat{X}) = cE(\hat{X}) = c\left(\frac{10}{11}\beta\right) = \beta$ hence $c = \frac{11}{10}$

$\frac{11}{10}\hat{X}$ is an unbiased estimator for β.

2. a) $Var(2\bar{X}) = 4 Var(\bar{X}) = 4\left(\frac{\sigma^2}{10}\right) = 4\left(\frac{\beta^2}{120}\right)$

b) $Var\left(\frac{11}{10}\hat{X}\right) = \left(\frac{11}{10}\right)^2 Var(\hat{X}) = \left(\frac{11}{10}\right)^2 \left[E(\hat{X}^2) - \left(\frac{10}{11}\beta\right)^2\right]$

$= \left(\frac{11}{10}\right)^2 \left(\frac{10}{12}\right) \beta^2 - \beta^2 = \frac{\beta^2}{120}$

$E(\hat{X}^2) = \int_0^\beta \left(\frac{10}{\beta^{10}} x^9\right) x^2 \, dx = \frac{10}{12} \beta^2$

$Var\left(\frac{11}{10}\hat{X}\right)$ is smaller - the estimate 5.587476 is likely better