CONFIDENCE INTERVALS

Video 1. Watch the introduction to interval estimation.

A. Let Z be a standard normal random variable. Find a number z such that $P(-z < Z < z) = 0.95$. You may want to use the R command `qnorm(·)`

B. Let $\overline{X}$ be the mean of a random sample of size 100 from a population with mean μ and standard deviation $\sigma = 2$ (note that this is the population standard deviation, not the standard deviation of $\overline{X}$). Substitute $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ into the expression in part A, then isolate μ to fill in the blanks:

$$P(\overline{X} - \underline{\qquad} < \mu < \overline{X} + \underline{\qquad}) = 0.95$$

C. Samples are taken and you find $\overline{x} = 7.767203$. Substitute this value in for $\overline{X}$ in part C to find the **95% confidence interval** for the population mean μ.

D. What's wrong with the expression $P(7.375203 < \mu < 8.159203) = 0.95$?

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E. Your 95% confidence interval is actually just the interval $(7.375203, 8.159203)$. What do these numbers mean? Try to give a non-technical explanation of the significance of this confidence interval.

Definition. If $\overline{X}$ is the mean of a random sample of size n (with n large) from a population with mean μ and standard deviation σ, then the $100(1-\alpha)\%$ confidence interval for μ is $\boxed{\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}$. Here $z_{\alpha/2}$ is the z-critical value: $P\left(Z > z_{\alpha/2}\right) = \alpha/2$ and can be found using the **R** command `qnorm(1` $-\ \alpha$`/2)`.

1. A random sample of 139 male house sparrows yields a sample mean blood plasma level (in pg/ml) of 209.46 and with a standard error of 16.62 (note that this is the standard error, not standard deviation). Calculate 95% and 99% confidence intervals for the true mean plasma level of male house sparrows.