**Math 321** **Exam 1 Formulas** **February 14, 2024**

| NAME: | SCORE: |
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**Instructions:** Solve $n-1$ the following $n$ problems and write your solutions in the space provided (or on additional paper, which is available at the front of the room). **You must choose a problem to skip: clearly mark that problem by writing SKIP or Xing out the space for that problem.** All solutions should include a clear method or argument and should use English words and sentences when appropriate. Clear and comprehensible solutions will generally earn more points than those that are hard to understand; a correct solution without supporting work may receive little or no credit. You may use a calculator, but phones and all other devices are forbidden. Answers may (and sometimes should) be left unsimplified.

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**Thoerem** (Bayes' law)**.** *If $A$ and $B$ are events with positive probability, then* $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

**Thoerem** (LoTP)**.** *If $A$ and $B$ are events and $0 < P(A) < 1$, then* $P(B) = P(B|A)P(A) + P(B|A^C)P(A^C)$

**Definition.** The **expected value** (or **mean**) of a discrete random variable $X$ with possible values $x_a, x_2, x_3, \ldots$ is $E(X) = \mu = \sum_i x_i P(X = x_i)$. The **variance** of $X$ is $\text{var}(X) = \sigma^2 = E\left[(X-\mu)^2\right]$.

**Definition.** The total number of successes in $n$ independent, identically distributed (iid) Bernoulli trials with parameter $p$ is a random variable with a **binomial distribution**. The PMF of a random variable $X$ having a binomial distribution with parameters $n$ and $p$ is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \ldots, n$$

The **R** syntax for the CDF is $P(X \le x) = \texttt{pbinom(x,n,p)}$.

**Proposition.** *The mean of a binomial distribution is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$.*

**Definition.** The number of trials until the first success in a sequence of independent, identically distributed (iid) Bernoulli trials with probability of success $p$ is a random variable with a **geometric distribution**. The PMF of a random variable $X$ having a geometric distribution with parameter $p$ is

$$P(X = x) = p(1-p)^{x-1} \text{ for } x = 1, 2, 3, \ldots$$

The CDF is $P(X \le x) = 1 - (1-p)^x$ for $x = 1, 2, 3, \ldots$.

**Proposition.** *The mean of a geometric distribution is $\mu = \frac{1}{p}$ and the variance is $\sigma^2 = \frac{1-p}{p^2}$.*

**Definition.** Suppose a sample of size $n$ is to be selected without replacement from a population of size $M_1 + M_2$ consisting of $M_1$ are successes and $M_2$ failures and the order of selection doesn't matter. When $n \le M_1$, the number of successes selected is a **hypergeometric** random variable and its PMF is

$$P(X = x) = \frac{\binom{M_1}{x}\binom{M_2}{n-x}}{\binom{M_1+M_2}{n}}$$

The **R** syntax for the CDF is $P(X \le x) = \texttt{phyper(x,M}_1\texttt{,M}_2\texttt{,n)}$.

**Proposition.** *The mean of a hypergeometric distribution is $\mu = n\left(\frac{M_1}{M_1+M_2}\right)$ and the variance is* $\sigma^2 = n\left(\frac{M_1}{M_1+M_2}\right)\left(\frac{M_2}{M_1+M_2}\right)\left(\frac{M_1+M_2-n}{M_!+M_2-1}\right)$.