EXAM 1 FORMULAS (PROPOSED)

Theorem (Bayes' Law)**.** *If A and B are events with positive probability, then*

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Definition. A **random variable** X assigns a number to each outcome in the sample space S.

(1) All random variables have a **cumulative distribution function (CDF)**: $F(x) = P(X \leq x)$.
(2) A discrete random variable has a **probability mass function (PMF)**: $m(x) = P(X = x)$.

Definition. The **expected value** (or **mean**) of a random variable X is $E(X) = \mu = \sum_x xp(x)$ (if X is a discrete RV with PMF $p(x)$). The **variance** of X is $\text{var}(X) = \sigma^2 = E\left[(X-\mu)^2\right]$.

Proposition. *For any random variable X, $var(X) = E(X^2) - [E(X)]^2$.*

Definition. The total number of successes in n independent, identically distributed (iid) Bernoulli trials with parameter p is a random variable with a **Binomial distribution**. The PMF of a random variable X having a binomial distribution with parameters n and p is

$$\boxed{b(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \ldots, n}$$

Proposition. *The mean of a binomial distribution is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$.*

Definition. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed (iid) Bernoulli trials, all with probability of success p. Let N be the trial on which the first success occurs. The random variable N is said to have a **geometric distribution** with parameter p and its PMF is

$$\boxed{g(n) = p(1-p)^{n-1} \text{ for } n = 1, 2, 3, \ldots}$$

Proposition. *The mean of a geometric distribution is $\mu = \frac{1}{p}$ and the variance is $\sigma^2 = \frac{1-p}{p^2}$.*

Definition. Suppose a sample of size n is to be selected without replacement from a population of size N, of which M_1 are successes. The number of successes selected is a **hypergeometric** random variable and its PMF is

$$\boxed{h(x) = \frac{\binom{M_1}{x}\binom{N-M_1}{n-x}}{\binom{N}{n}}}$$

Proposition. *The mean of a hypergeometric distribution is $\mu = n\frac{M_1}{N}$ and the variance is* $\sigma^2 = n\frac{M_1}{N}\left(1 - \frac{M_1}{N}\right)\left(\frac{N-n}{N-1}\right)$.

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