1. Suppose that we collect a random sample of size 9 from a population that is uniformly distributed on the interval \([0, \beta]\). This makes the probability density function of the population \(f(x) = \frac{1}{\beta}\) for \(0 < x < \beta\).

   a) Find a constant \(a\) such that \(a\bar{X}\) is an unbiased estimator of \(\beta\).

   b) Let \(Y\) be the largest number in the sample. Find the cumulative distribution function of \(Y\).

   c) Find a constant \(b\) such that \(bY\) is an unbiased estimator of \(\beta\).

   d) Calculate the variances of \(a\bar{X}\) and \(bY\) and use these numbers to decide which of the two unbiased estimators is better.

2. Let \(Z\) be a standard normal random variable.

   a) Find a number \(z\) such that \(P(|Z| < z) = 0.95\).

   b) Rearrange the inequality of part a to fill in the blanks in the following expression:

   \[ P(Z - \underline{\quad}) < 0 < Z + \underline{\quad} \approx 0.95 \]
3. Let $\bar{X}$ be the mean of a random sample of size $n = 9$ from a normally distributed population with mean $\mu$ (unknown) and standard deviation $\sigma = 12$. This means that $X$ is normally distributed with mean $\mu$ and standard deviation $\frac{\sigma}{\sqrt{n}} = 4$.

a) Using your work in problem 2 as a guide, fill in the blanks in the following expression:

$$P(\bar{X} - \underline{\quad} < \mu < \bar{X} + \underline{\quad}) = 0.95$$

b) Samples are taken and you find $\bar{x} = 50$. Substitute this value in for $\bar{X}$ in part a to find the 95% confidence interval for the population mean $\mu$.

c) What’s wrong with the expression $P(42.16 < \mu < 57.84) = 0.95$?

d) Your 95% confidence interval is actually just the interval (42.16, 57.84). What do these numbers mean?