EARTHQUAKES

Earthquakes occur pretty much at random, but at a rate we can estimate. For example, according to the US Geological Survey, the continental US had 3079 earthquakes of magnitude 4.5 or greater from Jan 1, 1920 to Dec 31, 2019. That's an average (mean) of 30.79 such earthquakes per year. However, the actual number of earthquakes is still unpredictable. The (random) number of earthquakes over a year, or any other time span, can be modeled as a Poisson random variable.

Definition. The PMF of a random variable X having a **Poisson distribution** with parameter $\lambda > 0$ is

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \dots$$

Proposition 1. If X has a Poisson distribution with parameter $\lambda > 0$, then $E(X) = \lambda$.

1. Let N_t be the number of earthquakes of magnitude 4.5 or greater in the continental US over t years. N_t should have a Poisson distribution with a parameter that depends on t.[1] For example N_1 is the number of significant earthquakes in one year, so it should have a Poisson distribution with $\lambda = 30.79$, the mean number of earthquakes per year.

a) What is the value of λ for N_2?

b) What is the probability that there will be two or more significant earthquakes before the end of the school year (round to 70 days)? First find the right value of λ, then use the PMF.

c) What is the value of λ for N_t (as a function of t measured in years)?

2. Let T be the **time** between significant earthquakes. Imagine starting a stopwatch after an earthquake and stopping it at the next earthquake; the measured time is T. This means that T is a random variable connected to the random variable N_t. Your goal in this problem is to use the known distribution of N_t to find a PDF for T.

a) Fill in the blank: $T > t_0$ if and only if $N_{t_0} =$ ______

b) Use your last answer to find the CDF for T by filling in the blanks:

$$\begin{aligned} P(T \leq t) &= 1 - P(T > t) \\ &= 1 - P\Big(N_t = \underline{\hspace{2cm}}\Big) \\ &= \underline{\hspace{4cm}} \text{ (use the Poisson PMF here)} \end{aligned}$$

Date: February 21, 2024.

[1]Technically, N_t is a family of random variables known as a **Poisson process**

c) How long until the probability of an earthquake occurring exceeds 0.5? (Use the CDF for T).

d) Differentiate the CDF you just found to find a PDF for T.

e) What is the mean wait time to the next earthquake? That is, find the expected value of T. It may help to recall integration by parts: $\int u dv = uv - \int v du$.

Challenge. Let T_2 be the time to the second earthquake. Find the probability density function of T_2 (use the same process as for the last problem).