SEQUENCES

Definition. The sequence $\{x_n\}$ converges to $x \in \mathbb{R}$ if $\boxed{\forall \varepsilon > 0\ \exists M \in \mathbb{N},\ n \geq M \implies |x_n - x| < \varepsilon}$

1. Suppose $\{x_n\}$ is a convergent sequence and $c \in \mathbb{R}$. Use the definition to prove that the sequence $\{cx_n\}$ converges. (This means finding the limit and proving that the sequence converges to that limit).

Proposition. *If $\{x_n\}$ is a convergent sequence such that $x_n \geq 0$, then* $\lim_{n\to\infty} x_n \geq 0$.

2. Find a convergent sequence $\{x_n\}$ such that $x_n > 0$ but $\lim_{n\to\infty} x_n = 0$.

Date: Due January 19, 2022.

Lemma. *A monotone sequence converges if and only if it is bounded.*

3. Let $c > 0$.

a) If $c < 1$, then $\lim_{n\to\infty} c^n = 0$. (Hint: use the lemma to prove convergence, then take the limit of both sides of the equation $c^{n+1} = c \cdot c^n$)

b) If $c > 1$, then $\{c^n\}$ is unbounded.